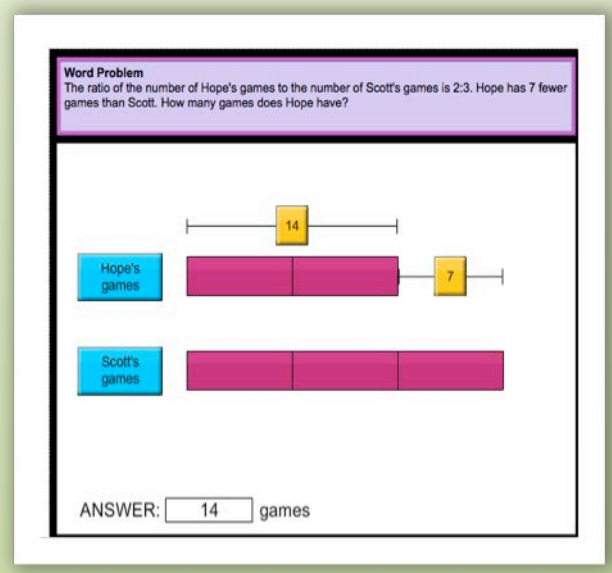
MATHEMATICAL REASONING AND REPRESENTATION

MODELING WITH TAPE DIAGRAMS

A RESOURCE GUIDE DEVELOPED BY TEACHERS FOR TEACHERS

Screen shot taken from thinkingblocks.com



COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

MEQUON-THIENSVILLE SCHOOL DISTRICT

CCLM
Common Core Leadership in Mathematics Project

Mequon-Thiensville School District

Tape Diagram
Teacher Resource

September 2012

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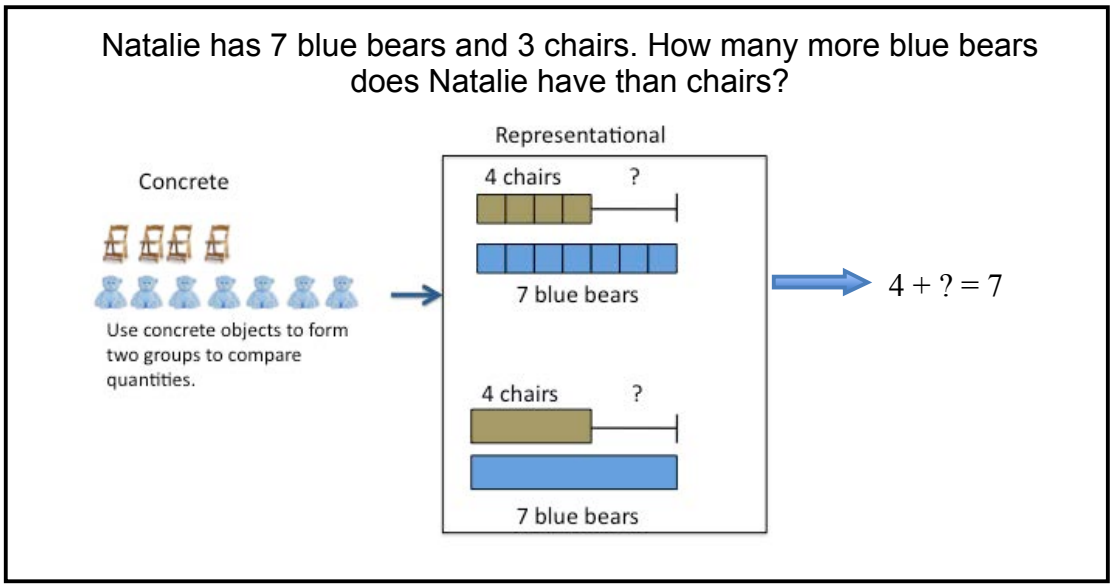
Tape Diagrams: The Missing Link

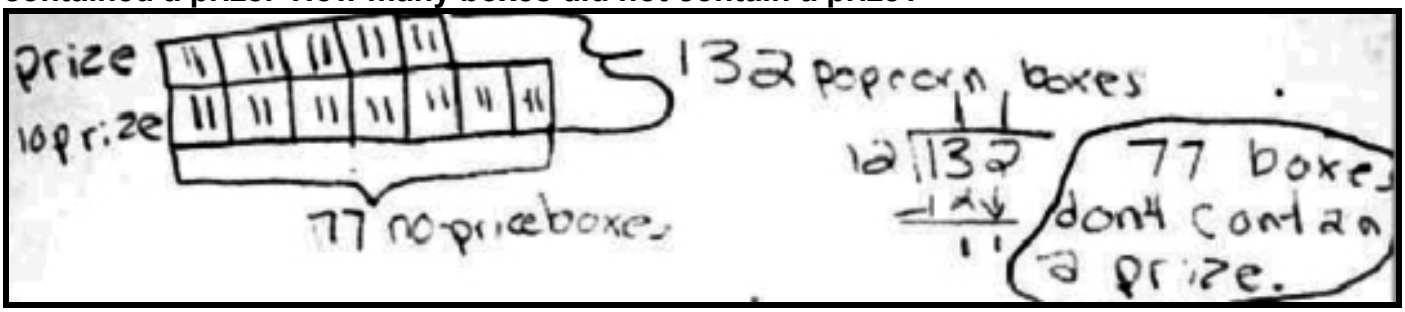
The Common Core Standards in Mathematics require students to know the "**why**" before the "**how**" and be *adept at modeling and defending their solutions* in conversations with their peers. One of the main emphases of these new, more rigorous standards is to require students to **visually represent their thinking** to help prove their solutions. This representation is the middle step of a powerful instructional sequence referred to as concrete-to-representational-to abstract and has been largely missing from mathematics teaching in the U.S.



This representational step or *visual model* is pulled throughout the Common Core Practice and Content Standards K-8. Tape diagrams are a powerful reasoning and organizational tool that give word problems a visual context. Perhaps most importantly, they lend themselves to work across the grade bands and throughout the mathematical domains. These models will provide your students with research-based Tier One instruction, which can easily be differentiated to meet varying student needs. Students will be able to tackle tough word problems with efficient and strategic models that put them on the road to algebra and future success in higher-level math.

If we want kids to start liking, learning, and understanding math, (and become more competitive with students internationally) we have to change how we teach math. We hope this compilation of tape diagrams will help teachers in the Mequon-Thiensville School District to do just that.

At the local fair, there were 132 boxes of popcorn on a shelf. 5 out of every 12 popcorn boxes contained a prize. How many boxes did not contain a prize?



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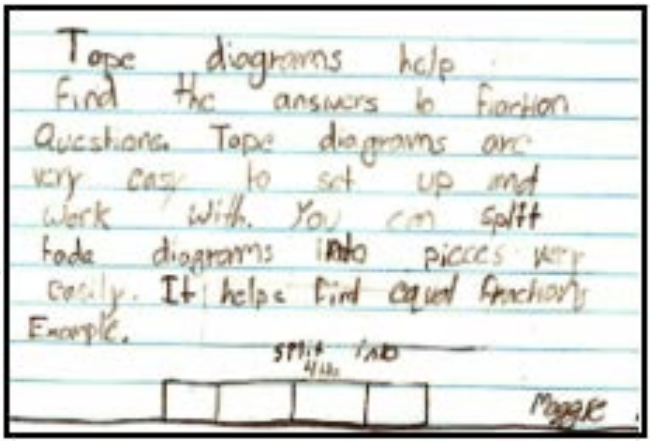
"The tape diagram model shows explicitly the problem structure, and the known and unknown quantities involved in a problem."

-- The Singapore Model Method for Learning Mathematics

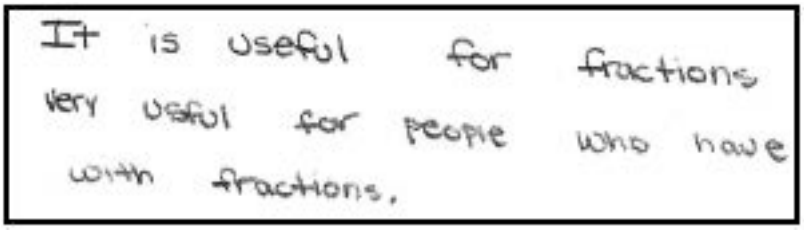
What We Love About Tape Diagrams!
Voices from MTSD Staff and Students

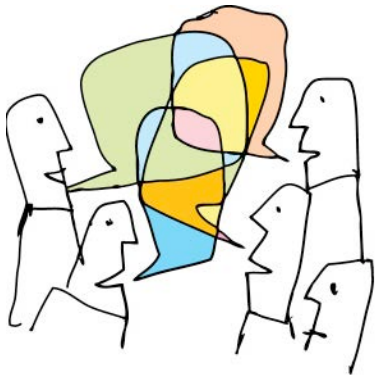
"Tape diagrams help me by breaking up the problems and doing them step by step. They help me visualize the problem so I can get a better understanding." –6th Grade Student

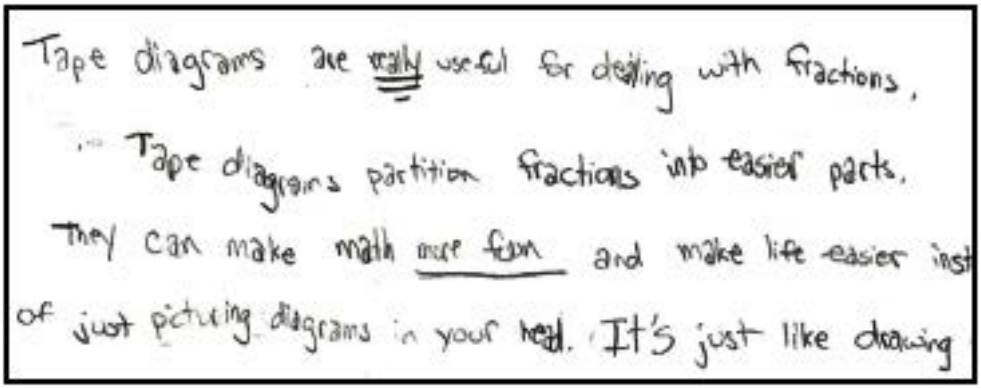

Tape diagrams help find the answers to fraction Questions. Tape diagrams are very easy to set up and work with. You can split tade diagrams into pieces very easily. It helps find equal fractions Example.

split 1/10 4th

Maggie

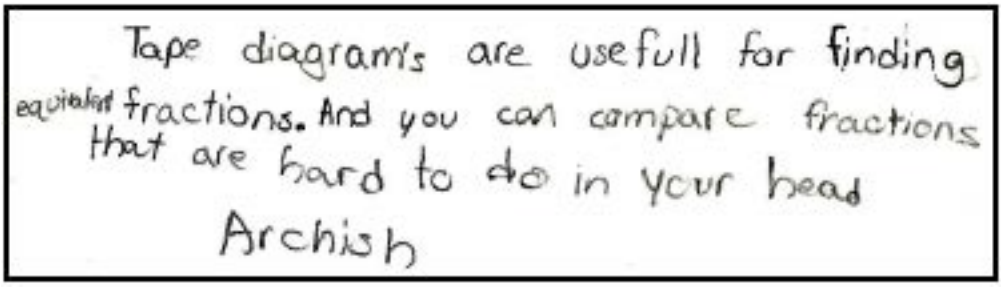

It is useful for fractions very useful for people who have with fractions.




Tape diagrams are really useful for dealing with fractions. Tape diagrams partition fractions into easier parts. They can make math more fun and make life easier inst of just picturing diagrams in your head. It's just like drawing

Tape diagrams have helped my students make the critical transition from concrete manipulatives to abstract work. Not only are my students more able to represent and *explain* their thinking verbally and in writing, they are also better equipped for confronting word problems and new problem types without prior instruction. ---MTSD 4th Gr Teacher

"Strip diagrams have helped me by making it easier to find a place to start on a problem. They have also helped me see the problem visually, which helps me solve it." --6th Gr. Student


Tape diagram's are usefull for finding equivalent fractions. And you can compare fractions that are hard to do in your head

Archish

Tape Diagram Resources

Web-based
Print
MTSD Teaching Staff

Web-based Resources

http://www.thinkingblocks.com
Students can build their own or teachers can use this interactive site on a SmartBoard.
Click on "modeling tool" to view videos.
Click on "start" button, then "problems" in bottom right corner for many, many problems of different types and skill levels.

http://www.risd.k12.nm.us/instruction/singaporemathbook.cfm
This site contains over 400 constructed response style word problems with answer keys and quizzes.

http://www.thedailyriff.com/articles/singapore-math-demystified-part-3-the-famous-bar-models-230.php
An informative article with video and sample problems.

http://www.youtube.com/watch?v=SC4OG11zOC8&feature=relmfu
A very well done overview of math common core standards in the middle school from the Teaching Channel

Singapore Math Bar Model Strategy
Google the above title for a pdf by Bill Jackson: Why Are Visualizations & Drawing As Important As Numbers?

Solving Algebra and Other Story Problems with Simple Diagrams: a Method Demonstrated in Grade 4–6 Texts Used in Singapore
Google the above title to find a landmark article by Sybilla Beckmann.

Print Resources

- Model Drawing for Challenging Word Problems: Finding Solutions the Singapore Way by Lorraine Walker (Grades 6-9)
- Step-by-Step Model Drawing: Solving Work Problems the Singapore Way by Char Forsten (Grades 3-6)
- Singapore Math 70 Must-Know Word Problems from Frank Schaffer Publications; available at many levels

MTSD Teaching Staff Resources

Lake Shore: Bridget Cuirlik (7th)
Katie Miszewski
Alice Merchant (Sp Ed)

Steffen: Mary Lock (8th)
Nancy Clark (6th)
Kaitlyn Albright (7th)

Donges Bay: Suzette Grub-Thur (5th)
Suzann Mattson (2nd)

Oriole Lane: Jodi Brekke (3rd)
Jane Malesweski (3rd)
Laura Kunz (K)
Sarah Krauss(K)

Wilson: Liz Cutter (4th)
Allison DeVillers (1st)
Jean Jones (1st)
Jenny Kim (4th)
Michelle Painter (4th)

District: Melissa Hedges
(K-8 Math Specialist)

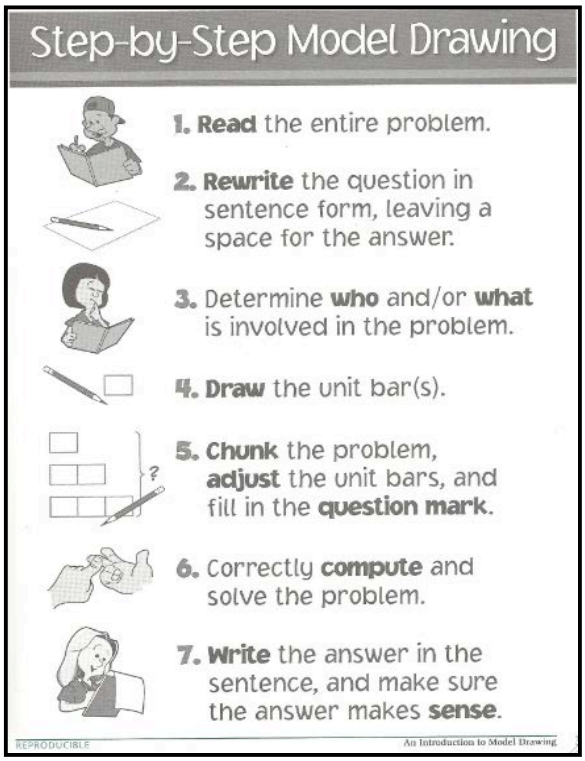
As you are getting started…

Tape Diagrams "are learning tools designed to help students solve math word problems accurately and efficiently." While drawing tape diagrams, "students model mathematical relationships and identify known and unknown quantities. The model provides students with a powerful image that organizes information and simplifies the problem solving process. By modeling increasingly complex word problems, students develop strong reasoning skills, which will facilitate the transition from arithmetic to algebra."

--Thinking Blocks Website http://www.thinkingblocks.com/
retrieved 9/2/2012

1) Use the first steps of the **Think-Aloud** process to introduce the story problem to your students:
   Step #1: Read the problem together
   Step #2: Students re-read the problem to each other
   Step #3: As a class, retell the problem *without talking about solution strategies.*
   Step #4: Ask students to visualize the problem. Acting it out is a great way to get this started.

2) After you have introduced your students using the 4 steps above**, follow the problem solving structure below.** (Omit "Read the entire problem.")



When many of us were first getting started with tape diagrams (model drawings) we appreciated the structure found to the left. You will see this format in some of the student work in this packet. A full-size copy of this document is found at the end of this packet.

3) **Start simple.** Don't go directly to challenge problems. Consider using http://www.thinkingblocks.com to get yourself and your students familiar with the process and structure of tape diagrams. Look at problems in this booklet that are a year or two below the grade you are teaching.

4) **Identify the problem type.** Different tape diagrams are used for different types of problems. For example, part-whole diagrams are used for Add to, Take from and Put Together Take Apart Problems. Comparison Tape Diagrams are used for Additive Comparison Problems. Multiplicative Comparison problems need a different type of tape diagram. See p. 38-39 of this resource book for a sample of some of these models.

5) **Model, model, model together** with your students…then have the students work problems on their own. We have found that many students need direct, interactive instruction using tape diagrams to solve problems. They also need numerous opportunities for guided practice before they work on their own. You may find the student work in this packet helpful as you work your way through various problems using tape diagrams.

6) As you are getting started with tape diagrams, first **solve the problem a few times yourself** talking through each step and practicing the language of tape diagram drawing. This would a **great thing to do during team meetings, PLCs, or content team meetings.** Places to go for good guided practice for teachers are found in this booklet as well as the resources mentioned below:
   • http://www.thinking blocks.com
   •Step-by-Step Model Drawing, Char Forsten (3th – 6th grade teacher resource book)
   •Model Drawing for Challenging Word Problems (6th- 9th grade teacher resource book)

7) **Provide graph/grid paper**. Students who have difficulty with spacing and the alignment of numbers can benefit from grid/graph paper to help organize their work. Grid/graph paper will also help align unit bars.

8) It's important that you and your **students label all unit bars clearly** and **label the unknown with a question**. Consider using different colors for different quantities.

9) **You will find that there are a variety of tape diagrams** that can be used to solve different types of problems (comparison, ratios, fractions, two unknowns, algebra etc.) See the Tape Diagram Models section of this resource for some of them!

10) Offer your students **short frequent practice –** Give students different types of problems with each unit so they can learn how to use this model to solve a variety of problems.

11) **White boards and colored markers** are a good way for students to show their work with peers. The different colors allow the students to identify the different parts of the diagram and show quantities being moved or changed. Different colors are also helpful when doing problems in notebooks.

12) Encourage your students to **write a written explanation** of how their tape diagram helped them solve the problem.

13) **Pretest and Posttest** before using tape diagrams. Give students a difficult word problem to solve in the pretest. Put that same problem on the posttest. They will be amazed at their ability to solve using a visual model. You will, too! ☺

Problems We've Tried with Our Students and They've Liked

(With Sample Student Solutions from Our Own MTSD Students)

Tape Diagram Problems Involving Algebraic Word Problems

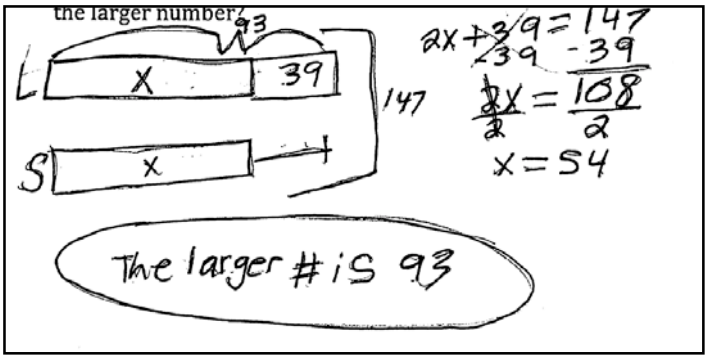
Appropriate for 8th Grade Pre-Algebra and Foundations

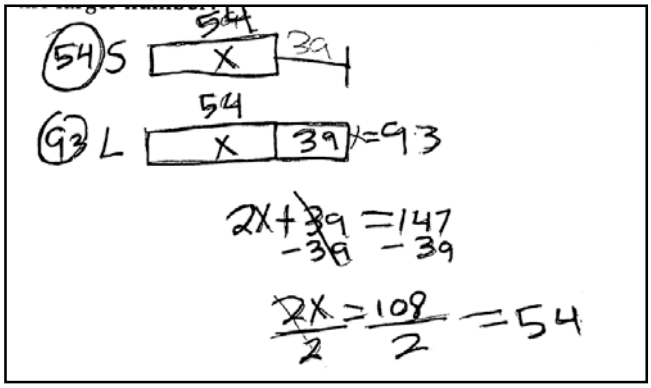
In using the tape diagrams with algebraic word problems, three distinct advantages surface:

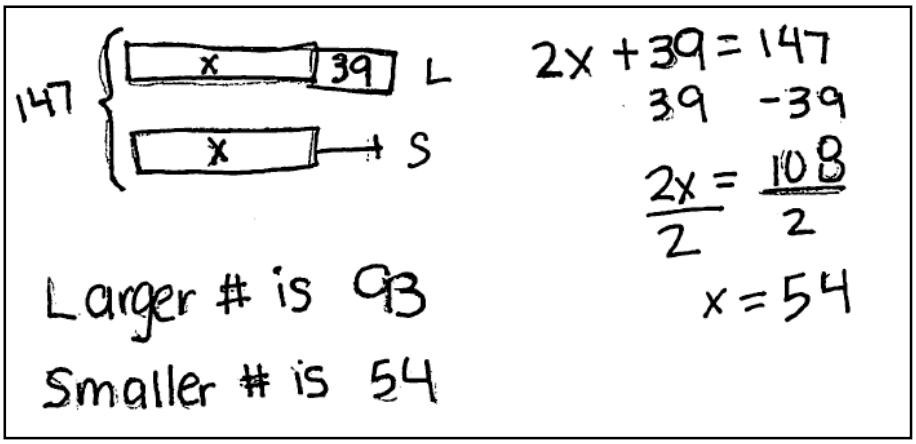
- It helps students “see” the interaction/relationship among quantities.
- It helps students construct algebraic equations.
- It helps students simplify, solve and check for reasonableness of the solution.

1. I am thinking of two numbers. The sum of my numbers is 147. The larger number is 39 more than the smaller number. What is the larger number?

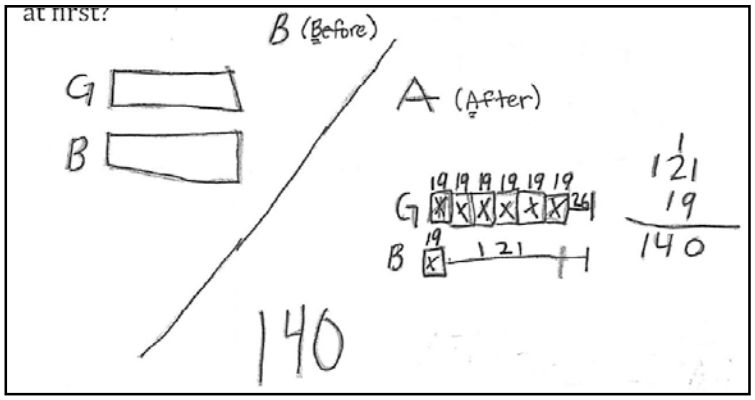
(Answer: 93) x + x + 39 = 147

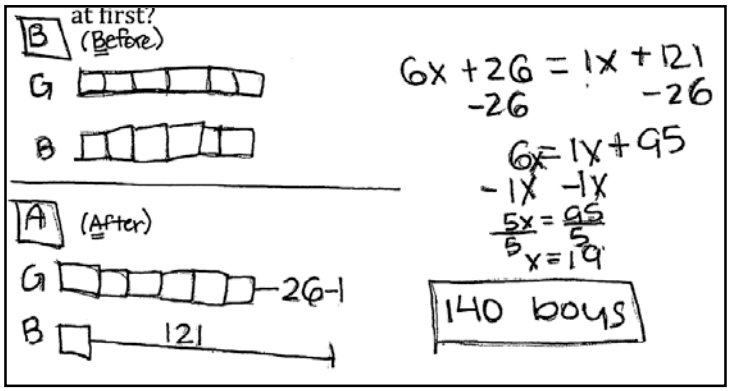




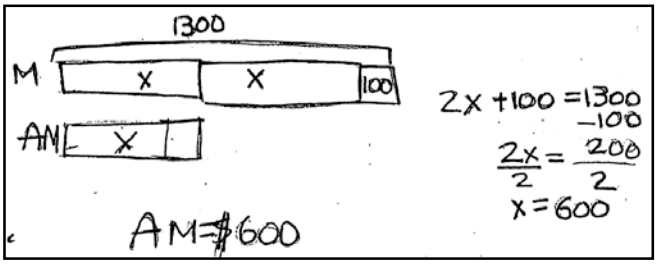


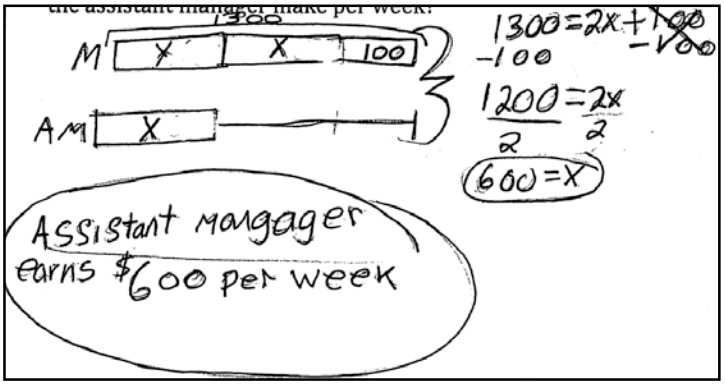
2. An equal number of girls and boys attended the magic show. When 26 girls and 121 boys left, there were 6 times as many girls as boys. How many boys attended the magic show at first? (Answer: 140 girls)



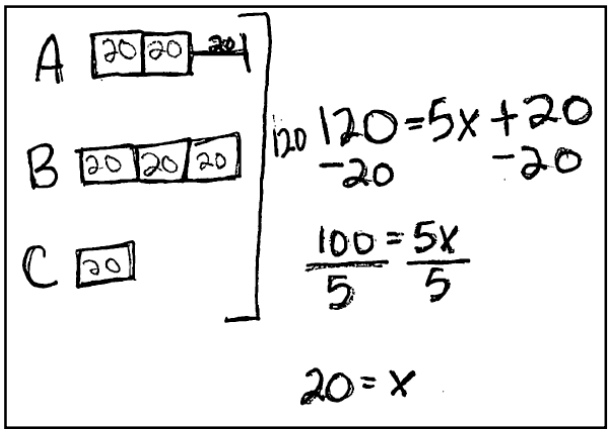


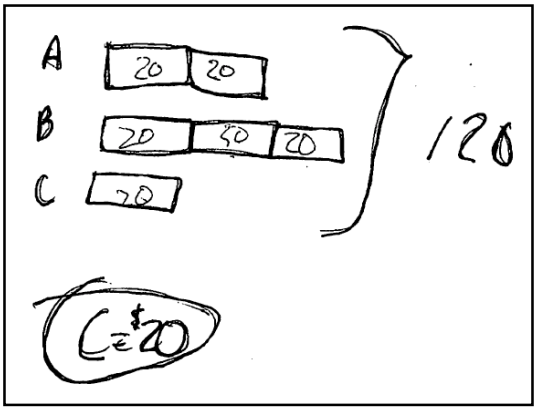
3. The manager earns $1300 a week, which is $100 more than twice what the assistant manager ears per week. What does the assistant manager make per week? (Answer: $600)



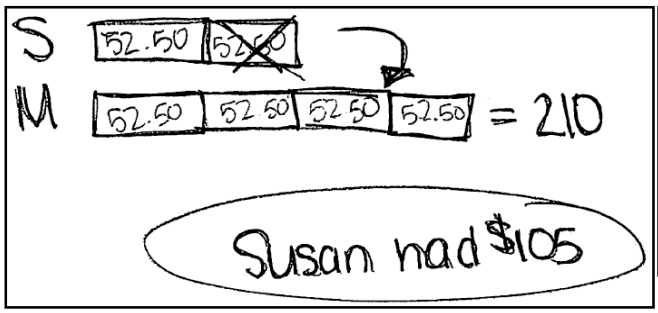


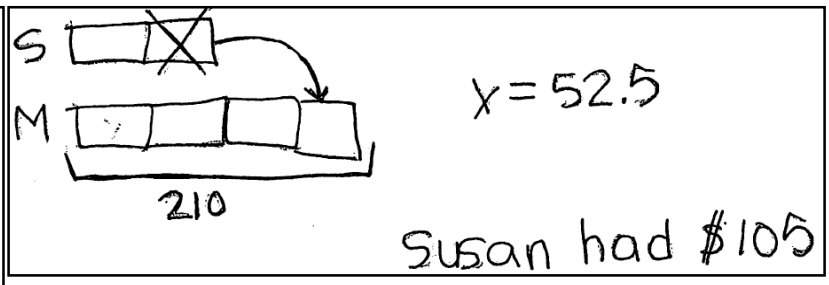
4. $120 is shared among 3 persons A, B, and C. If A receives $20 less than B, and B receives 3 times as much money as C, how much money does C receive? (Answer: $20)
(3x – 20) + 3x + x = 120



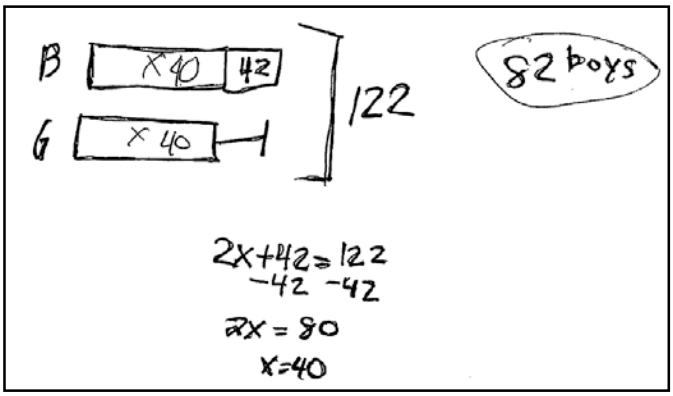


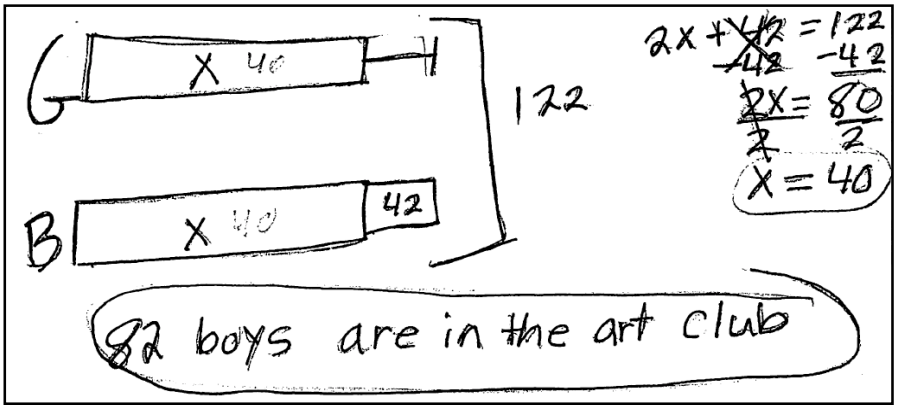
5. Susan had 2/3 as much money as Mary at first. After receiving ½ of Susan's money, Mary had $210. How much money did Susan have at first? (Answer: $105)
4x = 210 x = 52.5 2x = 105



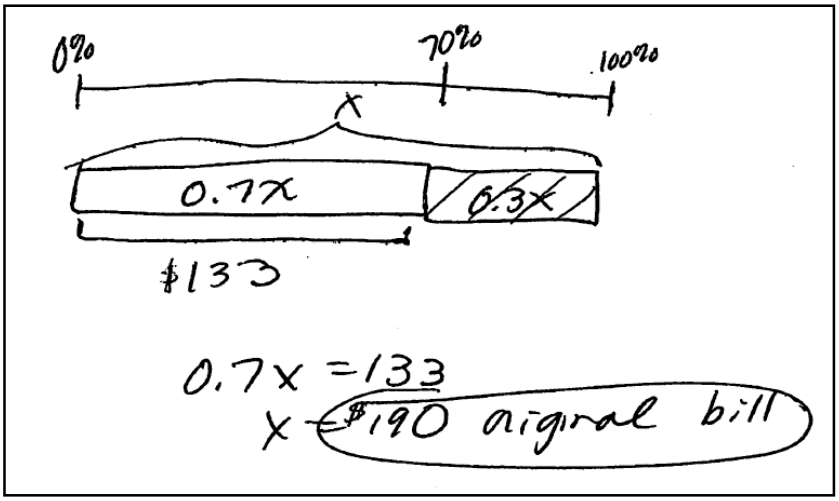


6. The art club has 122 members. There are 42 more boys than girls. How many boys are in the art club? (Answer: 82 boys) 122 = x + x = 42

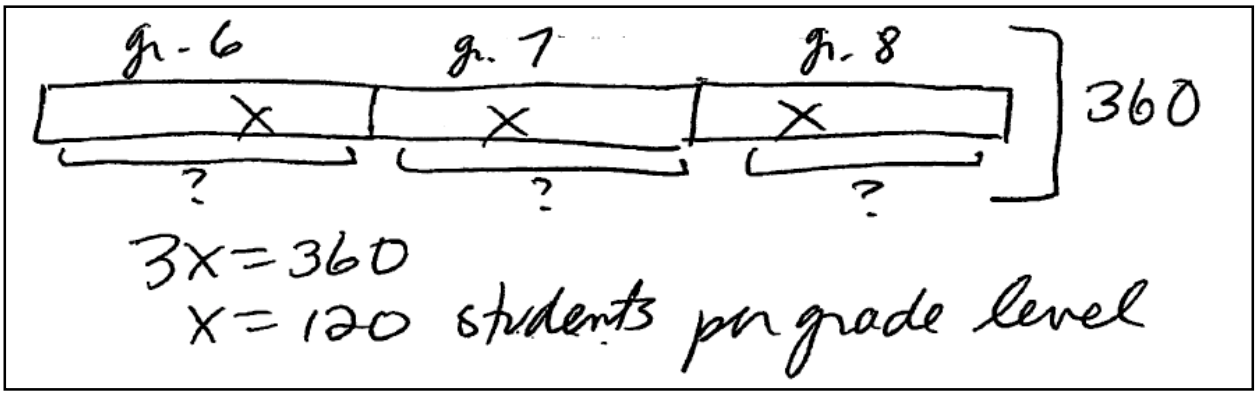




7. Mrs. Tan has a VIP card that entitles her to a discount of 30% off her dinner bill. If she pays $133 for her dinner, how much is the original bill? ($190) 0.7x = 133

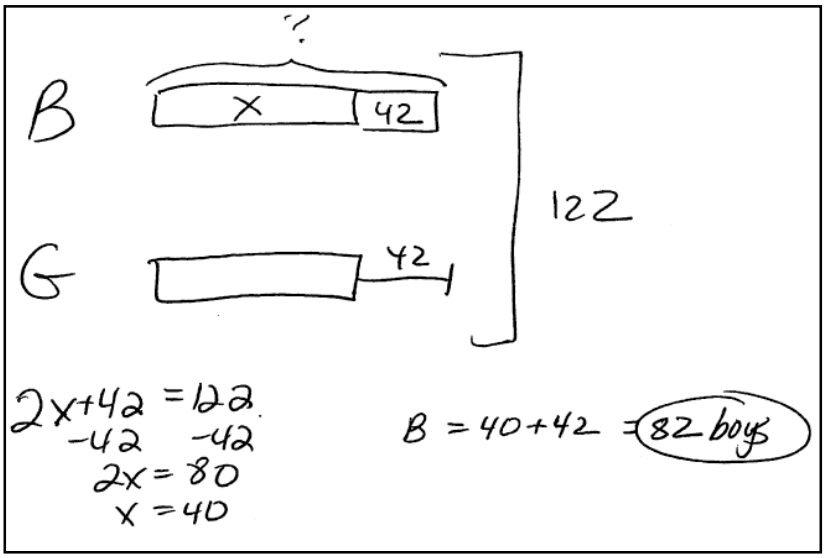


8. There are 360 students in a middle school. Each grade level has an equal number of students in grades 6,7 and 8. How many students are in each grade. 360 = 3x



9. Jake is 3 years older than Kyla and 2 years younger than Larry. The total of their ages is 41 years. Find Jake's age. (14 years)

41 + 3 – 2 = 42
3x = 42
x = 14

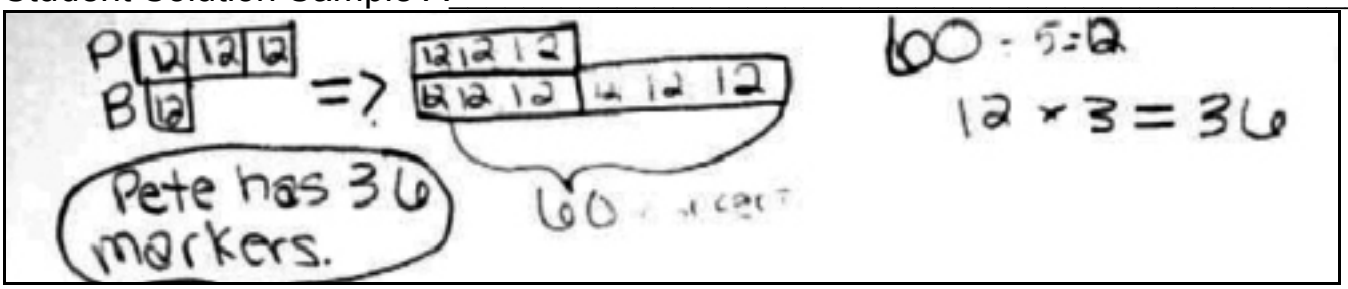


Tape Diagram Problems
Involving Ratio and Problem Solving

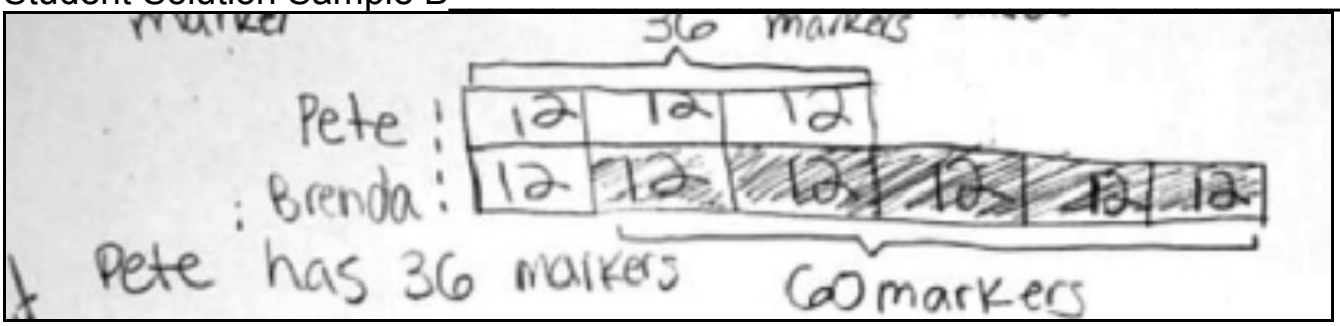
Appropriate for 6th and 7th Grade

1. The ratio of Pete's markers to Benda's markers was 3 to 1. When Brenda found 60 more markers, the ratio became 1:2. How many markers did Pete have?

Student Solution Sample A

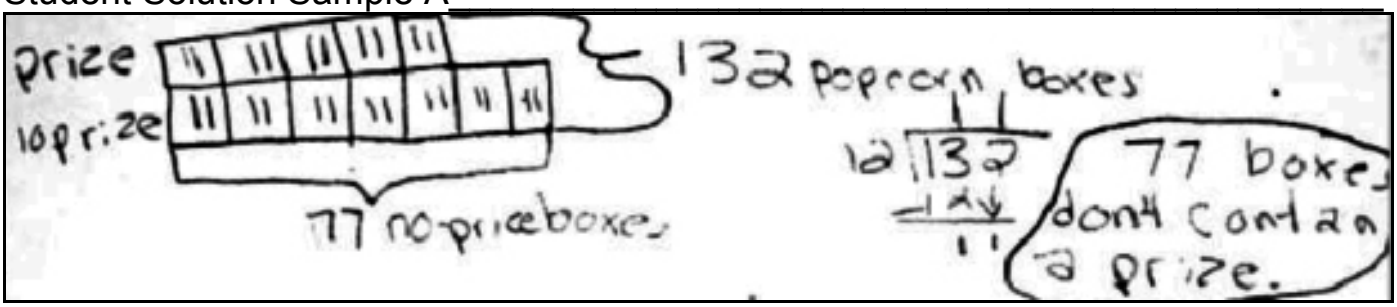


Student Solution Sample B

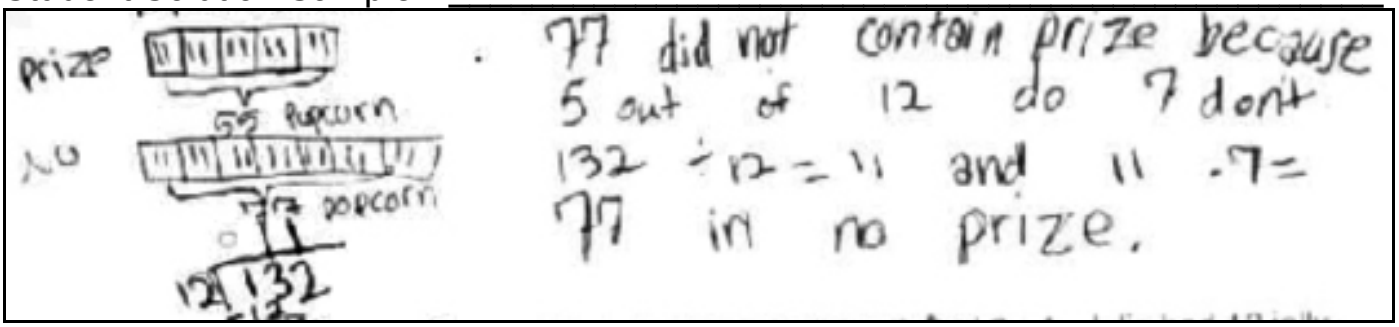


2. At the local fair, there were 132 boxes of popcorn on a shelf. 5 out of every 12 popcorn boxes contained a prize. How many boxes did not contain a prize?

Student Solution Sample A

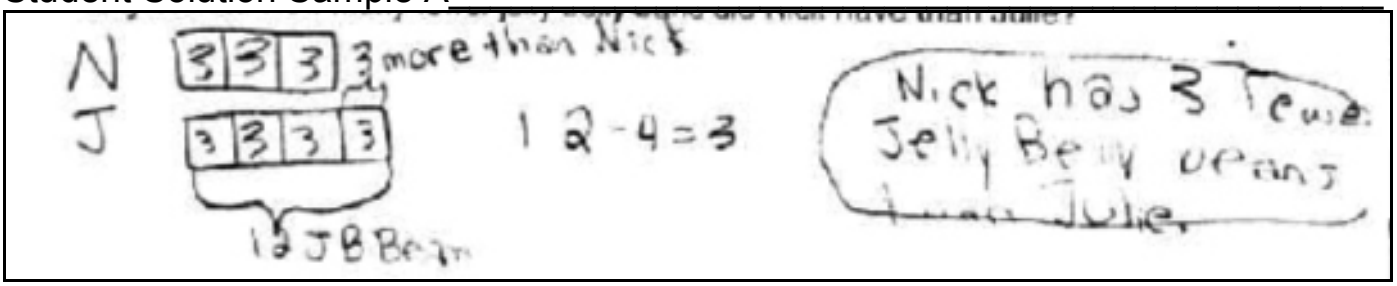


Student Solution Sample B

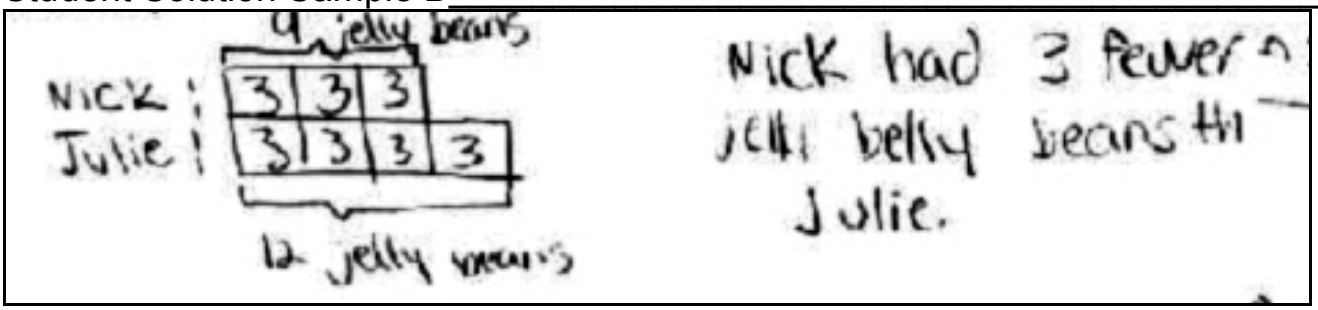


3. Nick and Julie shared some jelly belly beans in the ratio of 3:4. Julie had 12 jelly belly beans. How many fewer jelly belly beans did Nick have than Julie?

Student Solution Sample A

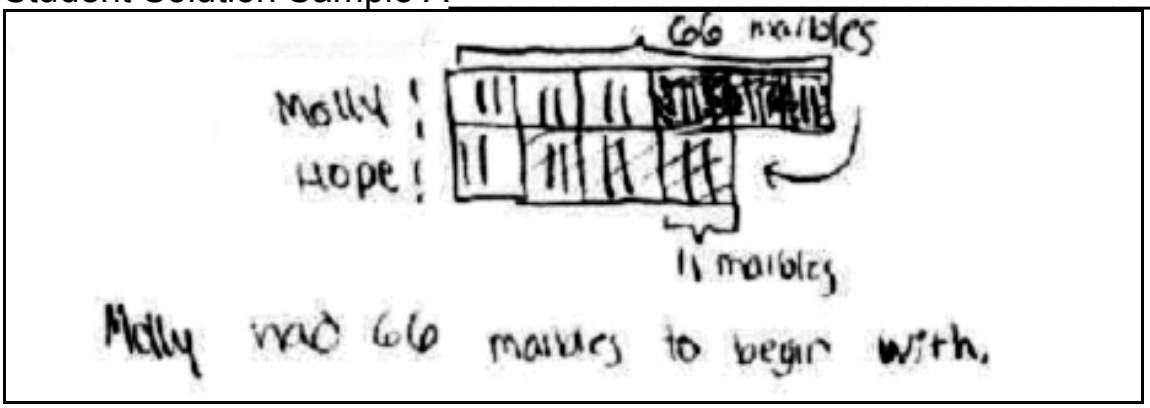


Student Solution Sample B

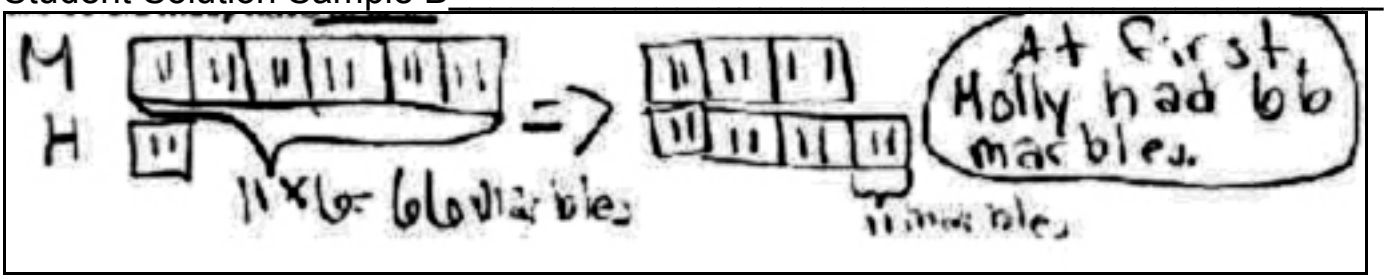


4. The ratio of marbles in Molly's bag to those in Hope's bag was 6:1. After Molly gave half of her marbles to Hope, Hope had 11 more marbles than Molly. How many marbles did Molly have at first?

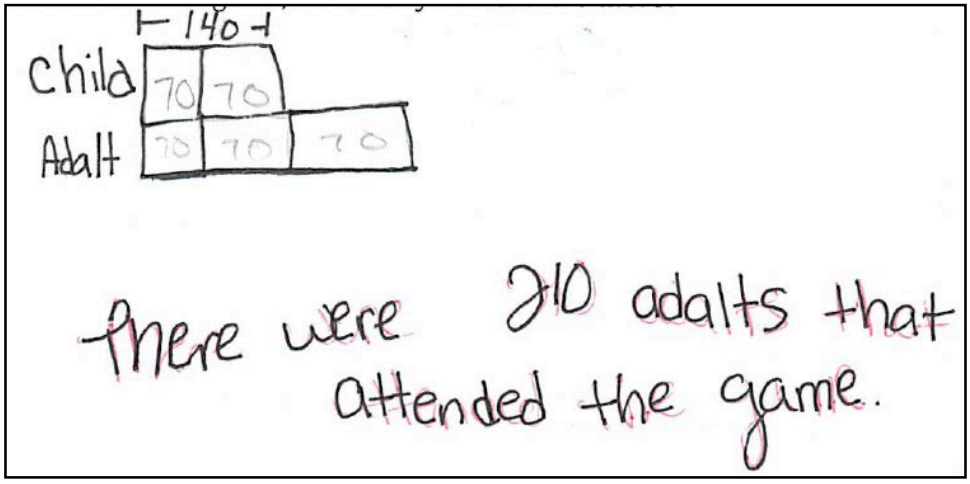
Student Solution Sample A



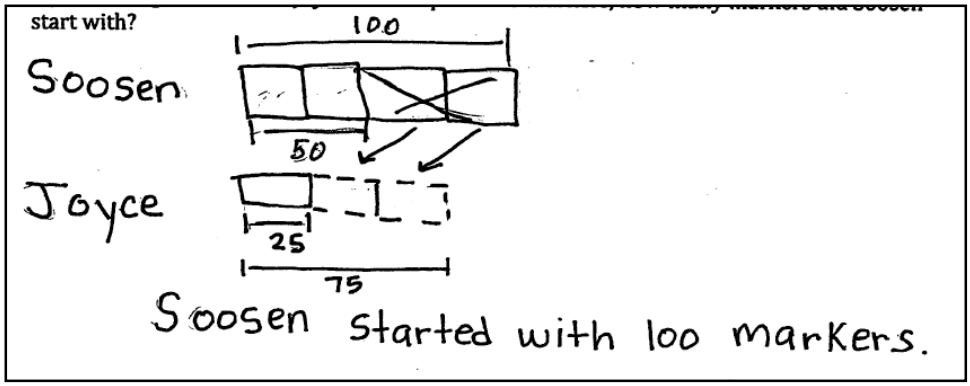
Student Solution Sample B

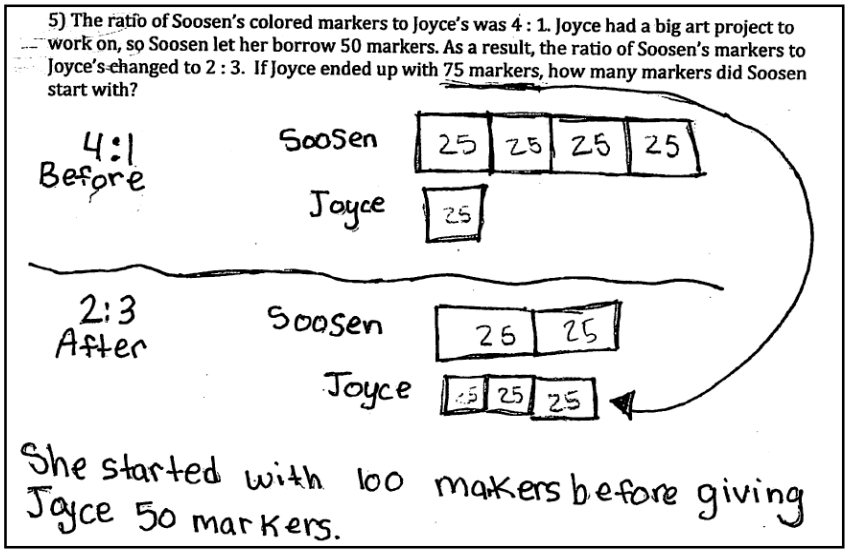


5. The ratio of children to adults at the football game was 2:3. If there were 140 children at the football game, how many adults were there?

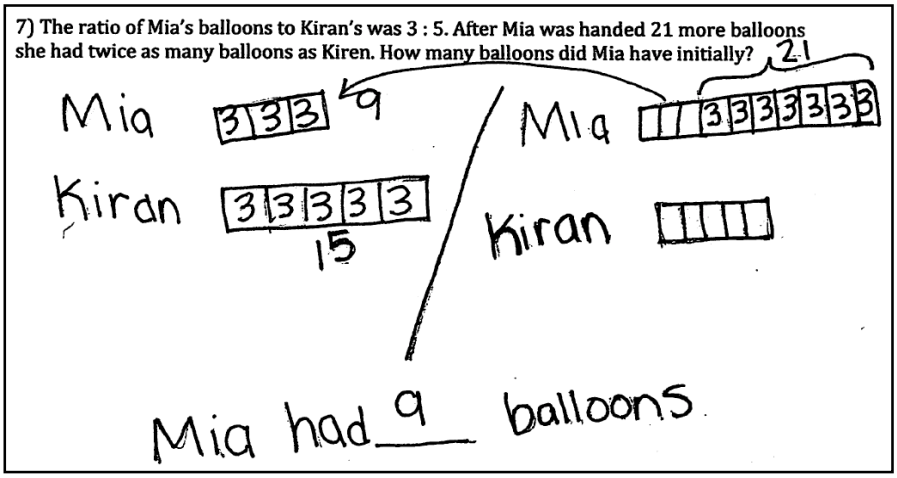


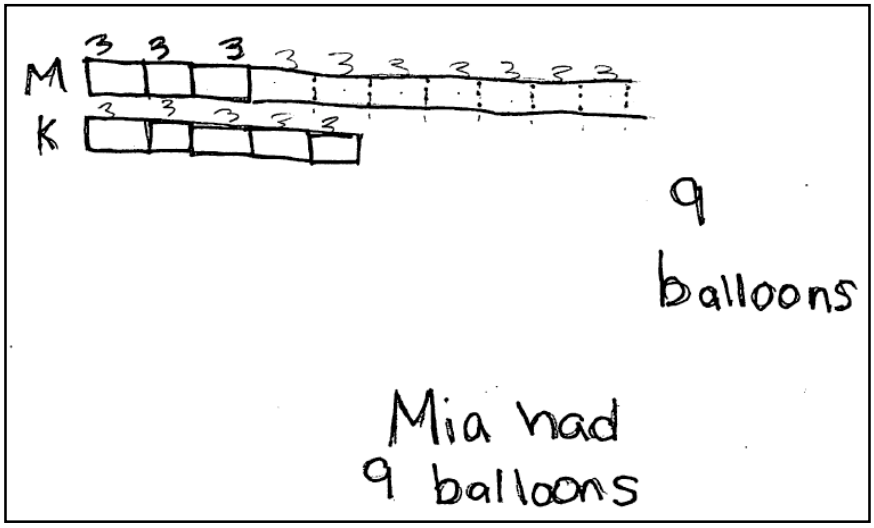
6. The ratio of Soosen's colored markers to Joyce's was 4 : 1. Joyce had a big art project to work on, so Soosen let her borrow 50 markers. As a result, the ratio of Soosen's markers to Joyce's changed to 2 : 3. If Joyce ended up with 75 markers, how many markers did Soosen start with?

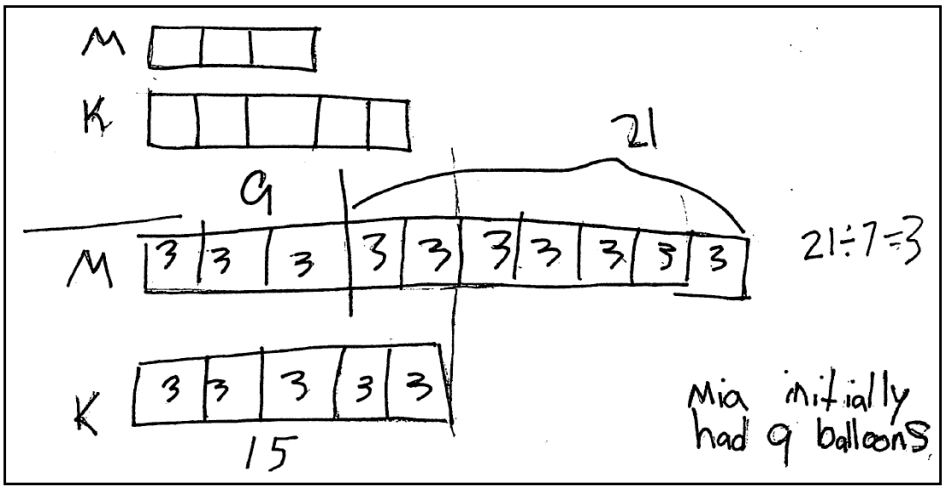




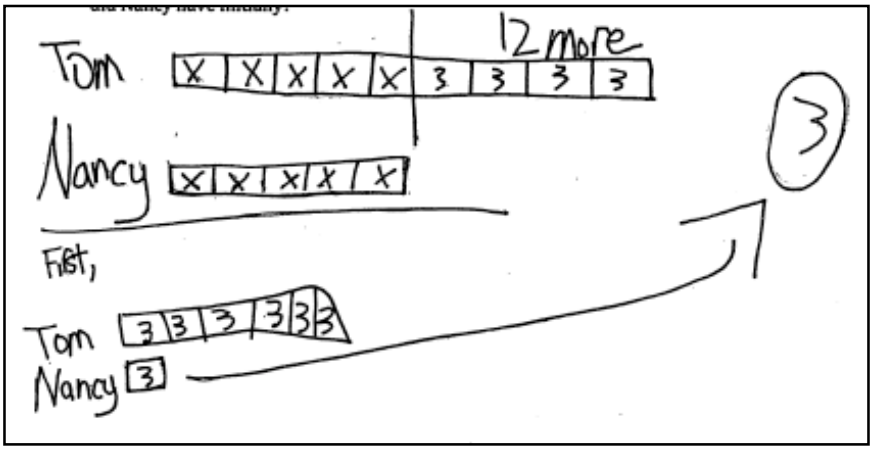
7. The ratio of Mia's balloons to Kiran's was 3:5. After Mia was handed 21 more balloons she had twice as many balloons as Kiren. How many balloons did Mia have initially?

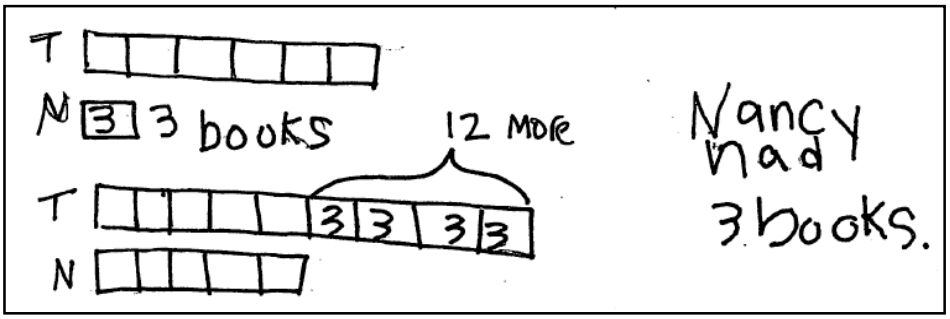




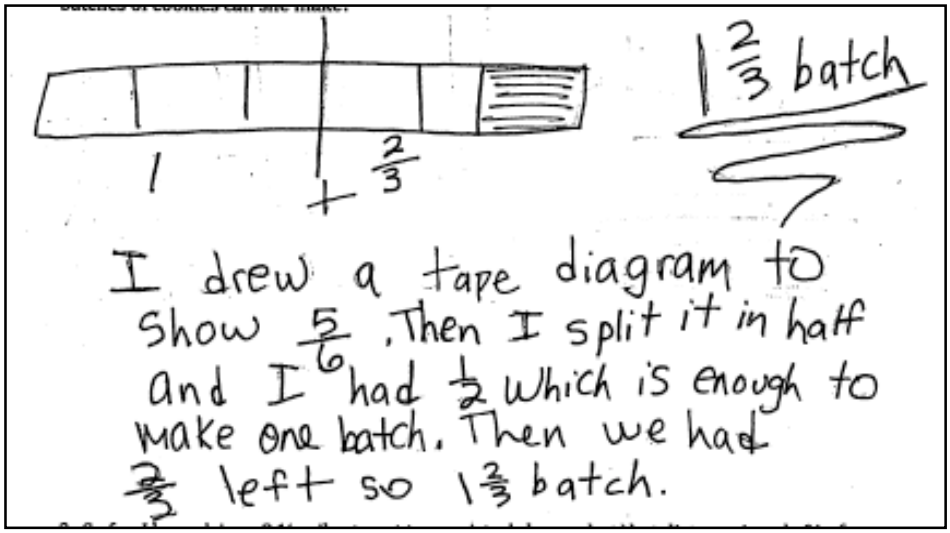


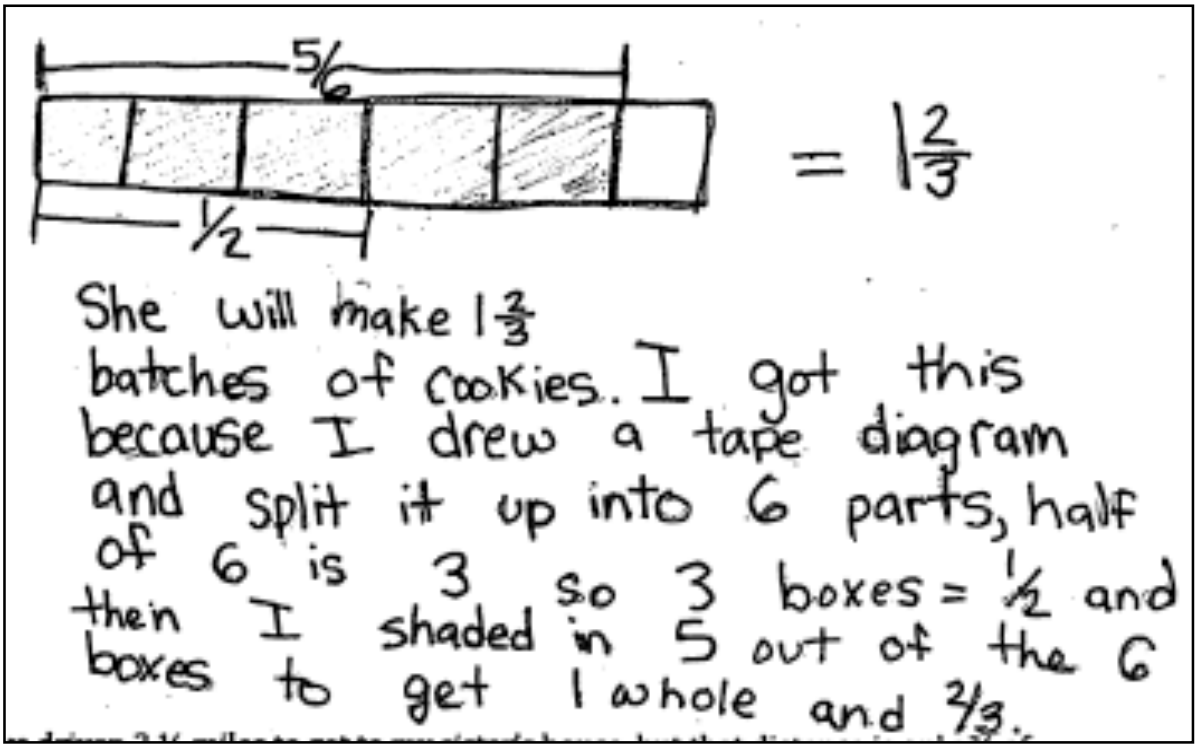
8. Tom and Nancy each collect Civil War books. Initially Tom had 6 times as many books as Nancy. After Tom gave Nancy some of his books for her birthday, the ratio of Tom's books to Nancy's was 9:5, and Tom had 12 more books than Nancy. How many Civil War books did Nancy have initially?





9. Maura has 5/6 cup of flour. She wants to make cookies and reads from the recipe that each batch of cookies requires ½ cup flour. How many batches of cookies can she make?

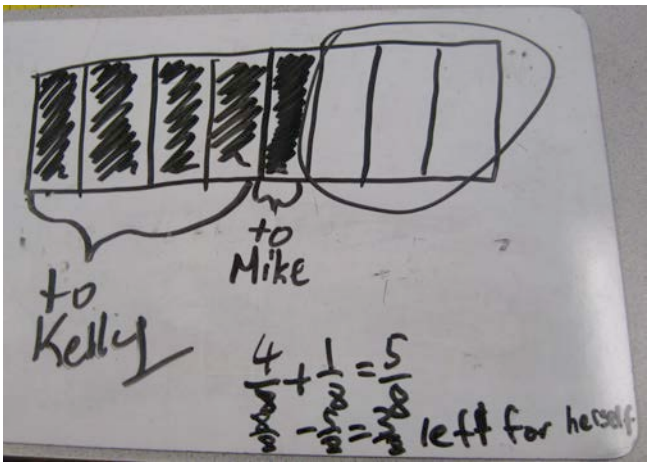


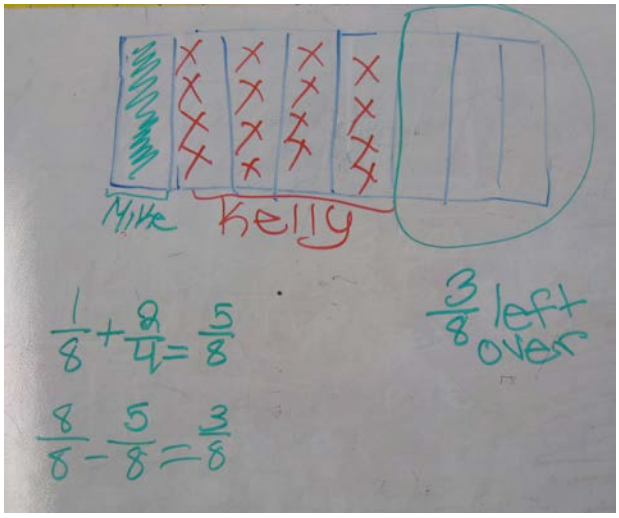


Tape Diagram Problems Involving Fractions and Percents

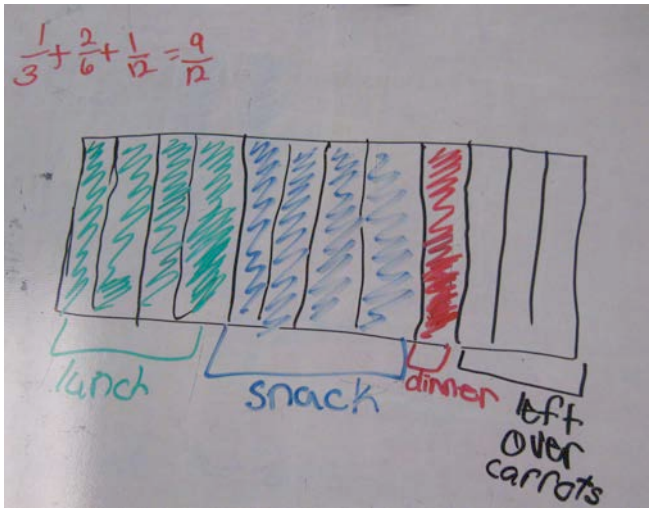
Appropriate for 4th-5th Grade Students

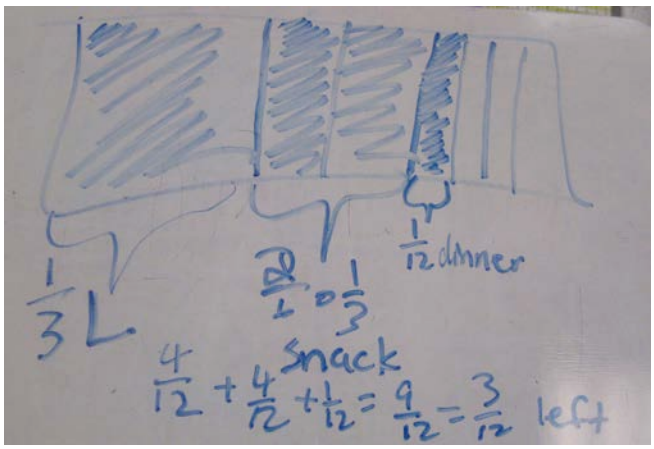
Zaya is moving to another town and decides to give 2/4 of her book collection to her friend Kelly and 1/8 of her book collection to her friend Mike. How much of her book collection does Zaya have left?"



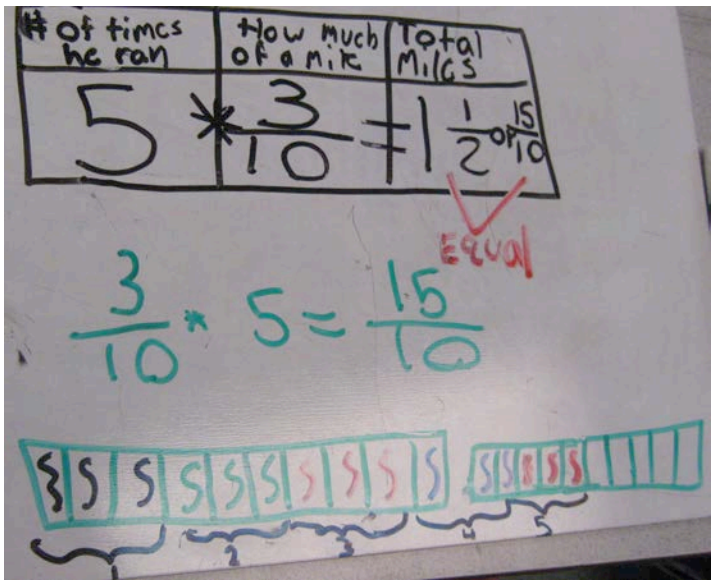


Ms. C buys a bag of carrots. She eats 1/3 of the bag for lunch, 2/6 as a snack and 1/12 at dinner. How much of the bag of carrots is left at the end of the day?

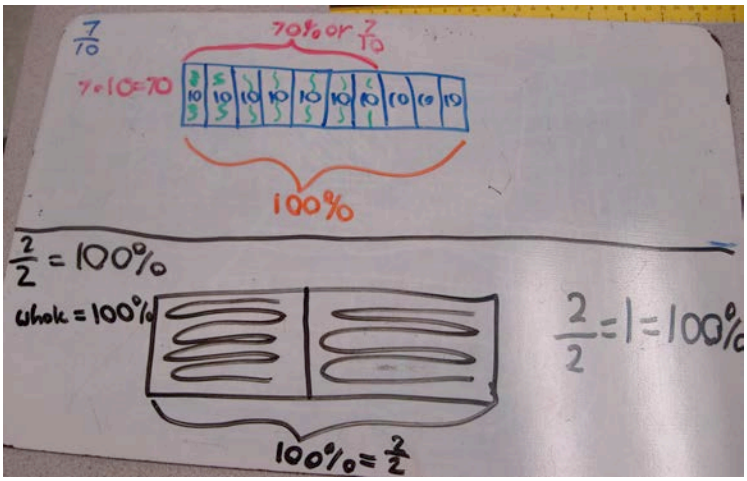


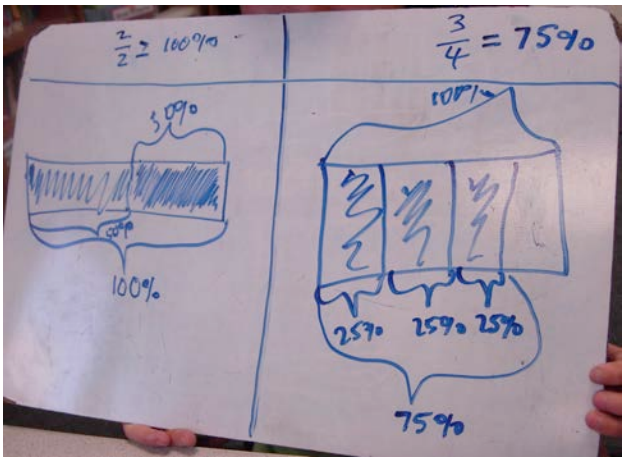


Michael is running laps around his school's gym. Each lap is 3/10 of a mile. If Michael runs 5 laps, how much of a mile will he have run?

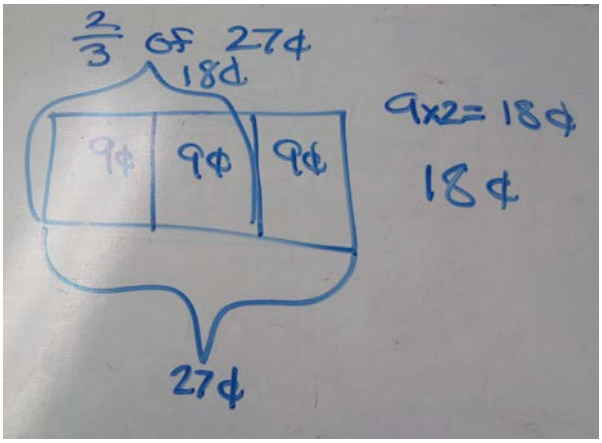


Demonstrate visually that 3/4 is equal to 75 percent. 2/2 is equal to 100 percent. 7/10 is equal to 70 percent.

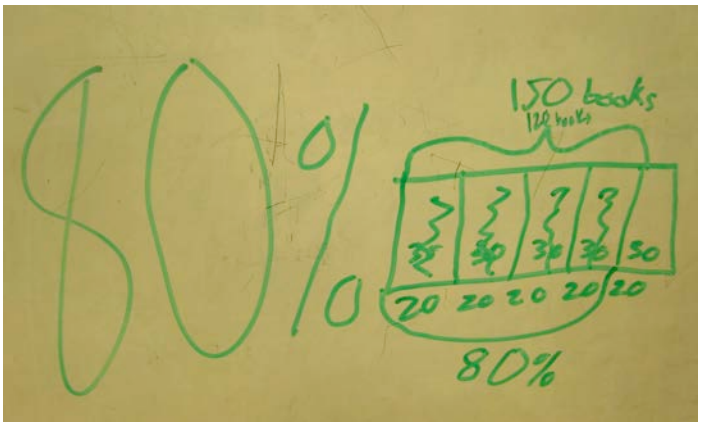


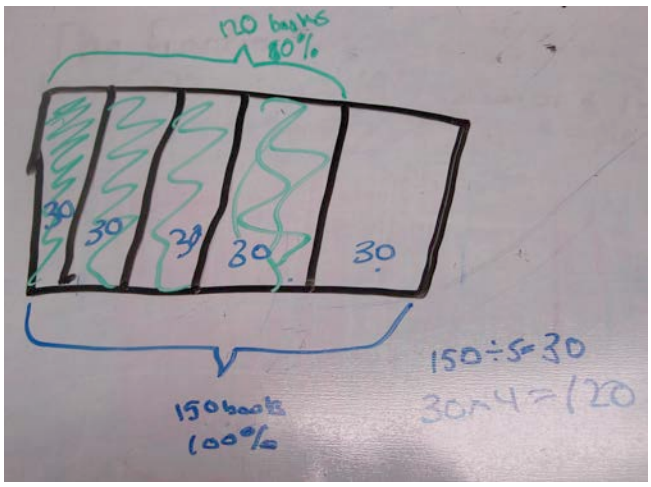


What is 2/3 of 27 cents?



Ms. C is putting books away in her classroom library. So far she has shelved 120 of a total of 150 books. What percentage of the total books has she shelved thus far?



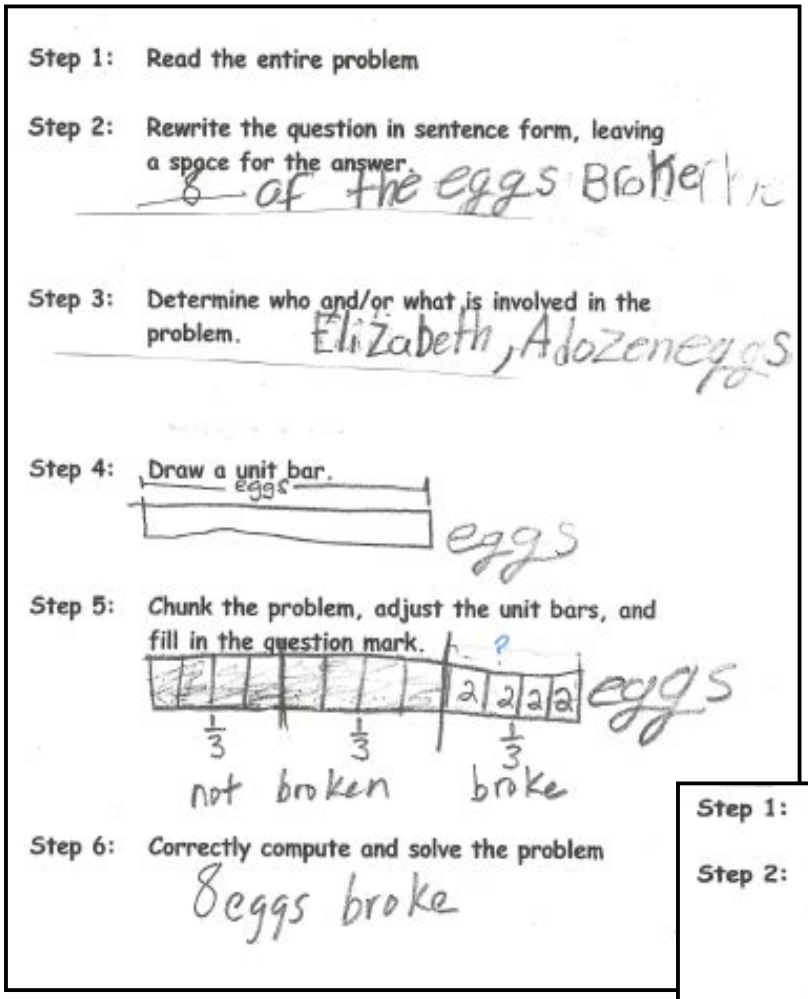


Tape Diagram Problems Involving Fractions, Multiplication, and Multiplicative Comparison

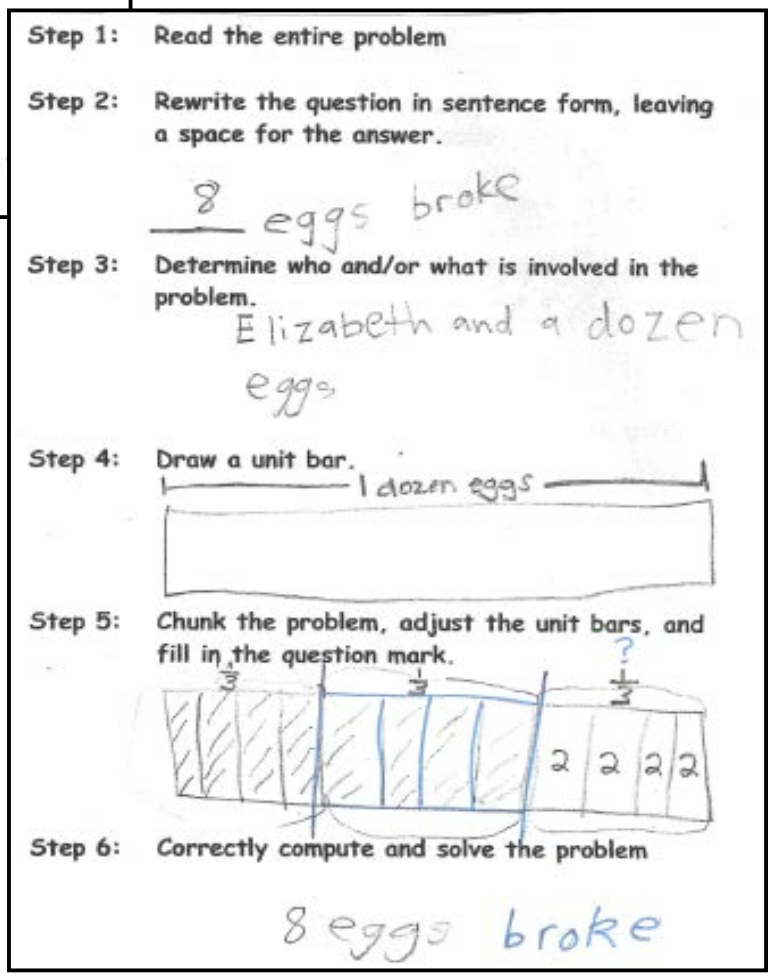
Appropriate for 3rd-4th Grade Students

Elizabeth bought a dozen eggs. She dropped her bag on the way home, and 2/3 of the eggs broke. How many eggs broke?

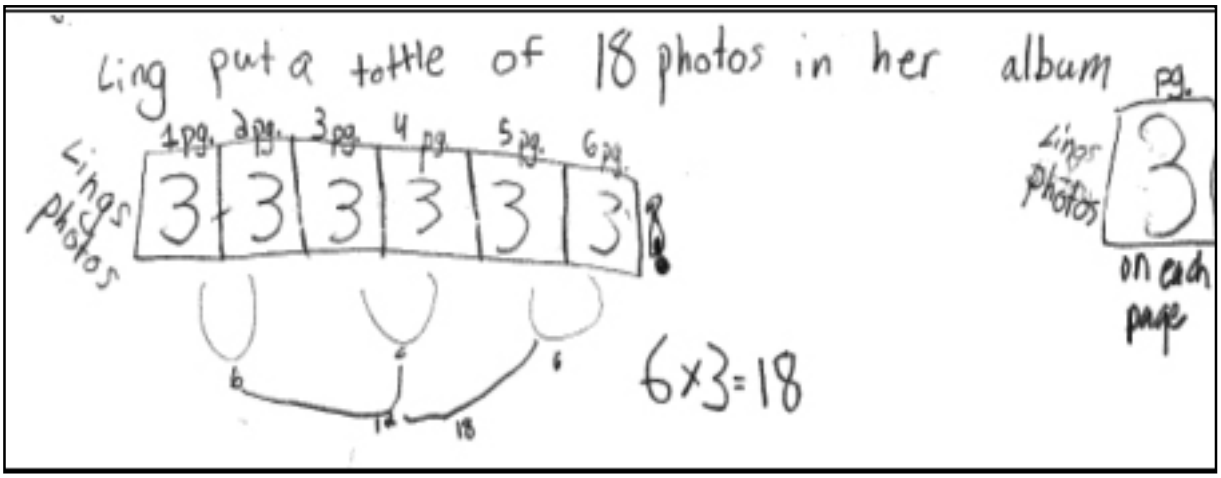
Student Solution Sample 1



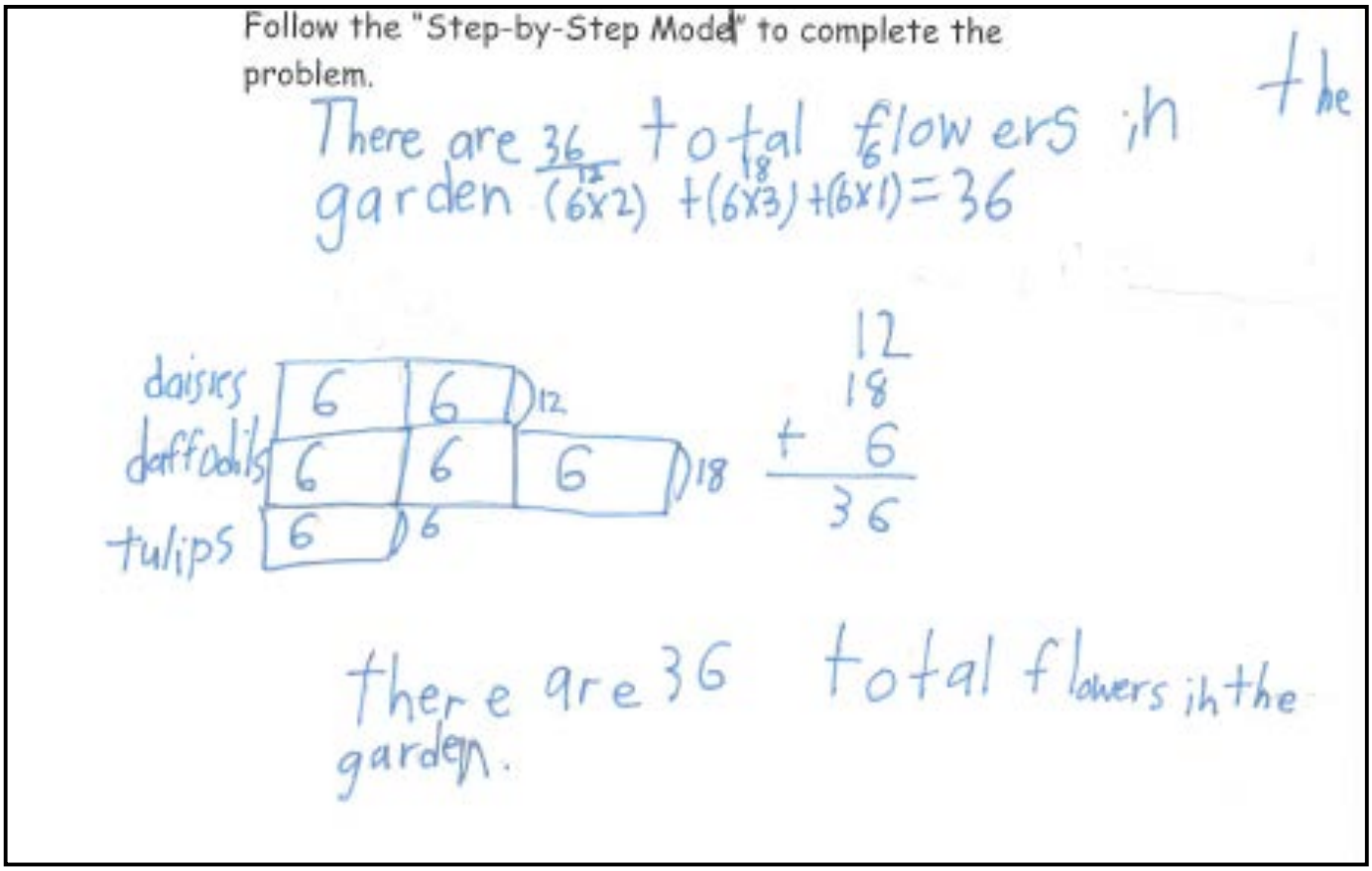
Student Solution Sample 2



Ling wants to put some photos in her photo album. She has 6 pages in her album. She puts 3 photos on each page. How many photos does she put in her album?

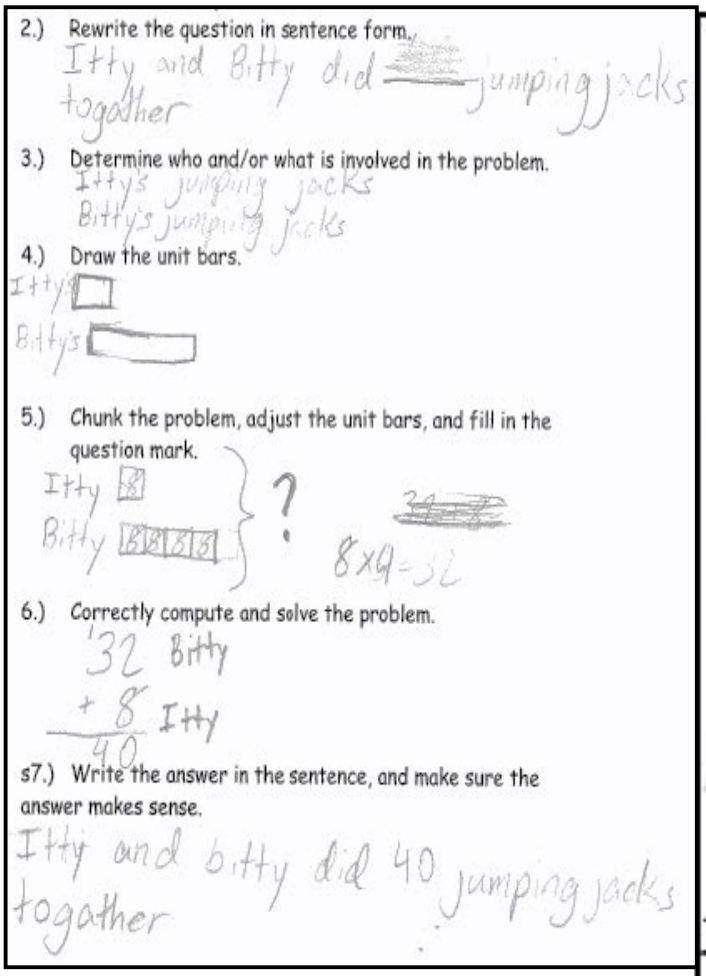


There are tulips, daisies, and daffodils growing in the garden. There are 2 times as many daisies as tulips and 3 times as many daffodils as tulips. If there are 12 daisies, how many total flowers are in the garden?

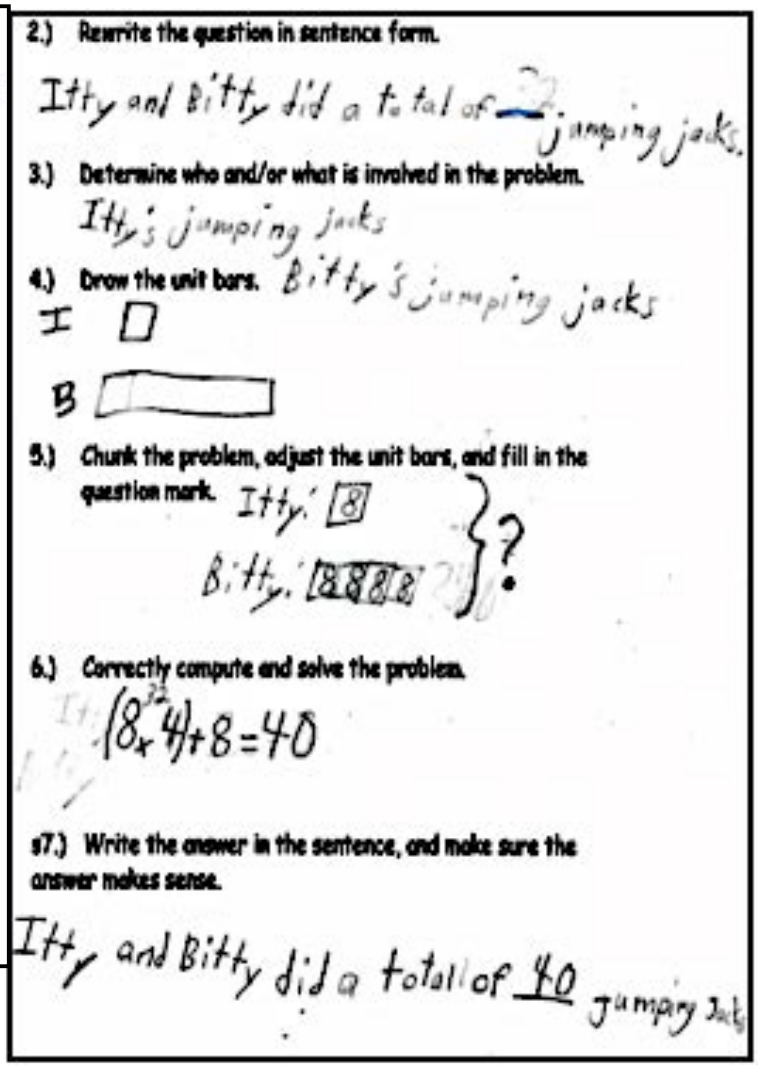


Itty and Bitty each did jumping jacks. Itty did 8 jumping jacks. Bitty did 4 times as many jumping jacks as Itty. How many jumping jacks did Itty and Bitty do together?

Student Solution Sample 1

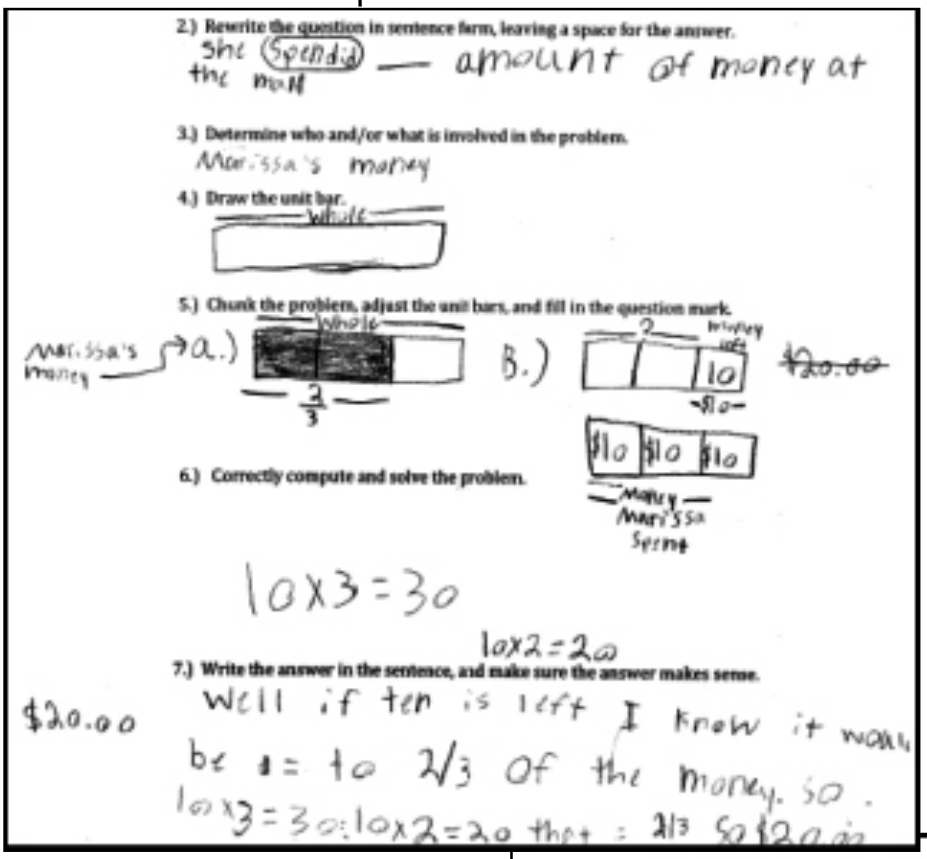


Student Solution Sample 2

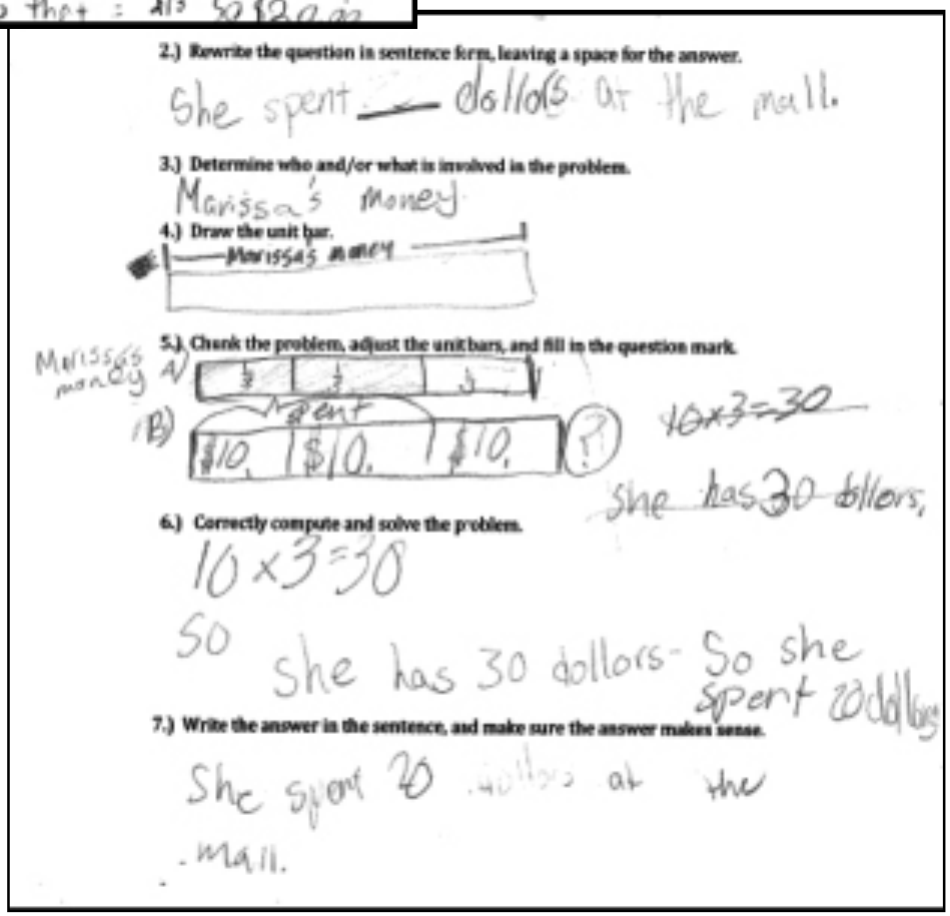


Marissa spent 2/3 of her money at the mall and had $10.00 left. How much money did she spend at the mall?

Student Solution Sample 1

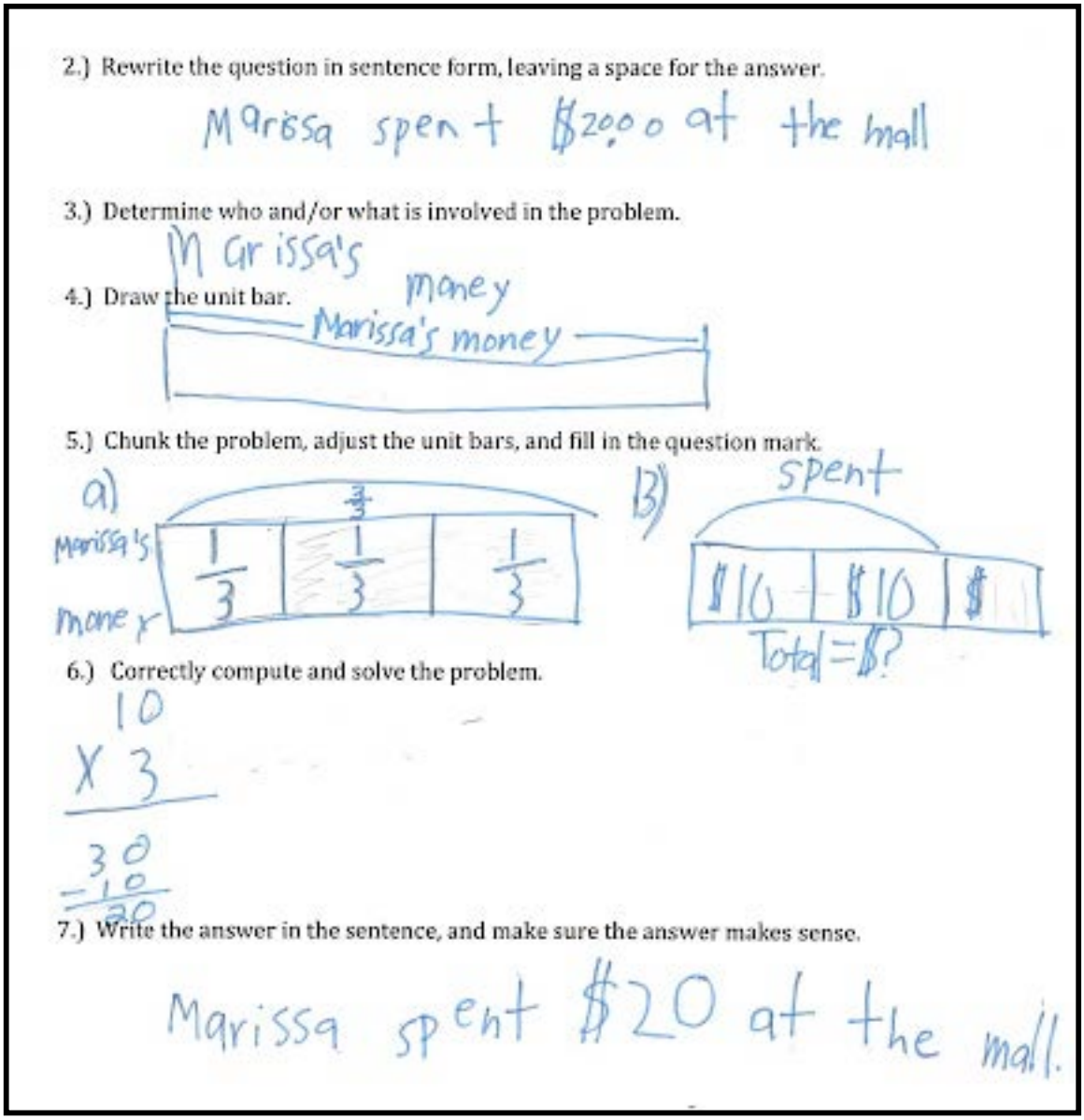


Student Solution Sample 2



Marissa spent 2/3 of her money at the mall and had $10.00 left. How much money did she spend at the mall?

Student Solution Sample 3



2.) Rewrite the question in sentence form, leaving a space for the answer.

Marissa spent $20.00 at the mall

3.) Determine who and/or what is involved in the problem.

Marissa's money

4.) Draw the unit bar.

Marissa's money

5.) Chunk the problem, adjust the unit bars, and fill in the question mark.

a)

Marissa's money

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

b)

spent

$10 $10 $

Total = $?

6.) Correctly compute and solve the problem.

$$\begin{array}{r} 10 \\ \times\ 3 \\ \hline 30 \\ -10 \\ \hline 20 \end{array}$$

7.) Write the answer in the sentence, and make sure the answer makes sense.

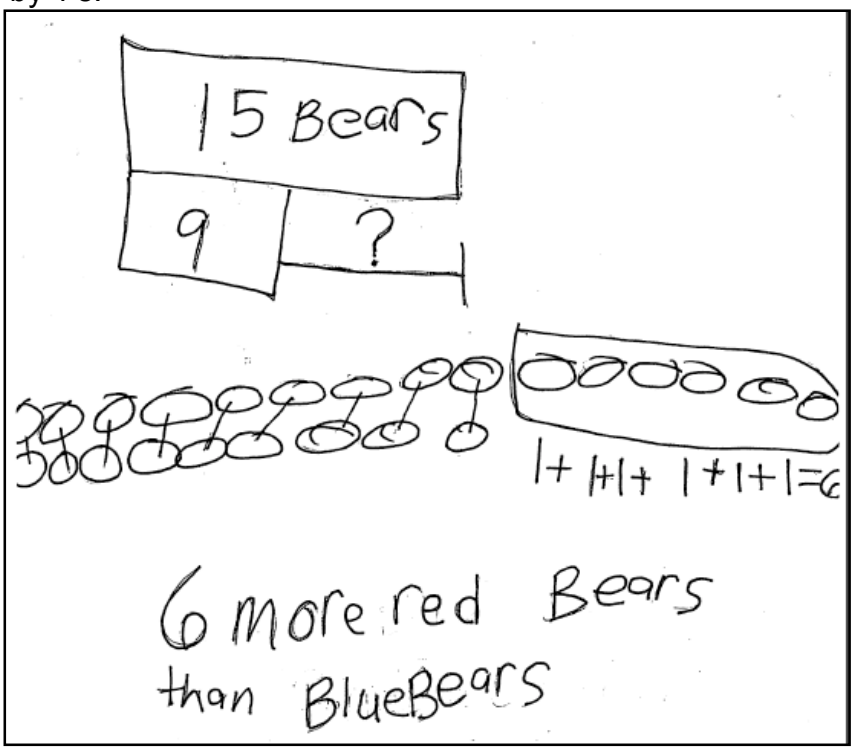
Marissa spent $20 at the mall.

Compare Problems
Addition and Subtraction

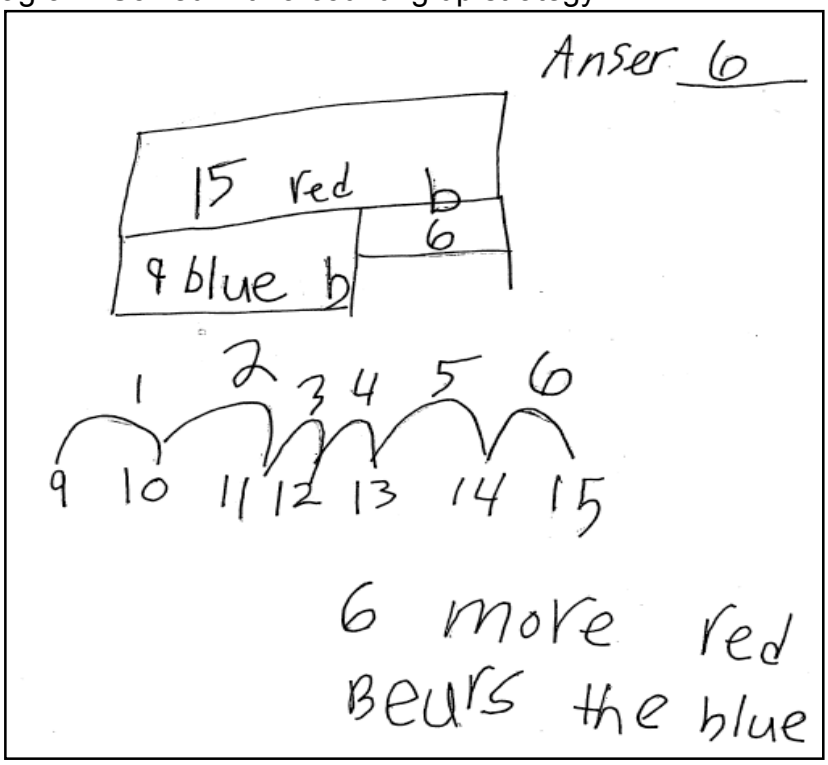
1st and 2nd Grade

Clare has 15 red bears and 9 blue bears. How many more red bears than blue bears does Clare have?

Discrete tape diagram paired with a continuous tape diagram. Solved using a matching strategy and counting red bears by 1's.



Continuous tape diagram. Solved with a counting up strategy.



Tape Diagram Models

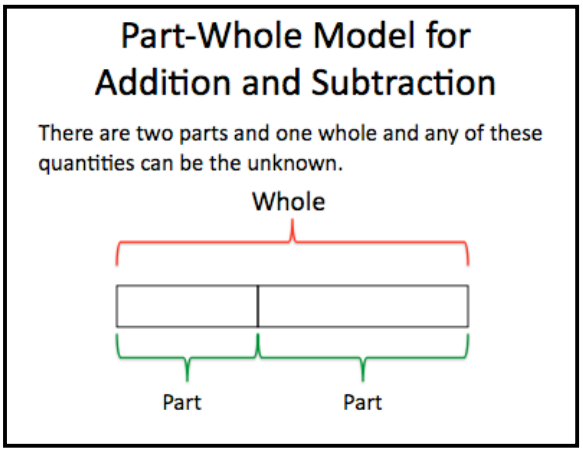
Tape Diagram - A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model, visual fraction model. (CCSSM Glossary)

"Tape diagrams provide a visual tool that enables students to determine what operation (addition, subtraction, multiplication, or division) to use to solve the problem. "

--The Singapore Model Method for Learning Mathematics

Tape Diagram Models and Problem Structures of Basic Word Problems

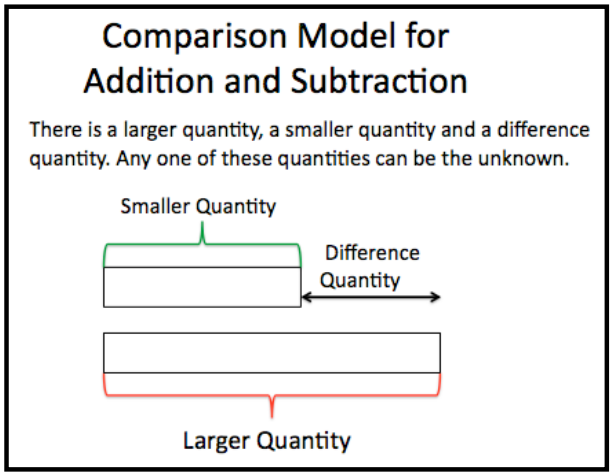
Type 1



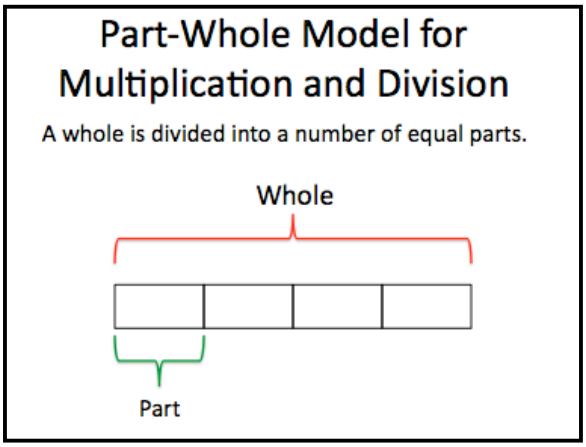
The Part-Whole Model is used with Add to, Take From and Put Together/Take Apart Problem Types. Either part, both parts or the whole can be unknown.

Type 2

The Comparison Model is used with Compare problems. The larger, smaller or difference quantity can be unknown.



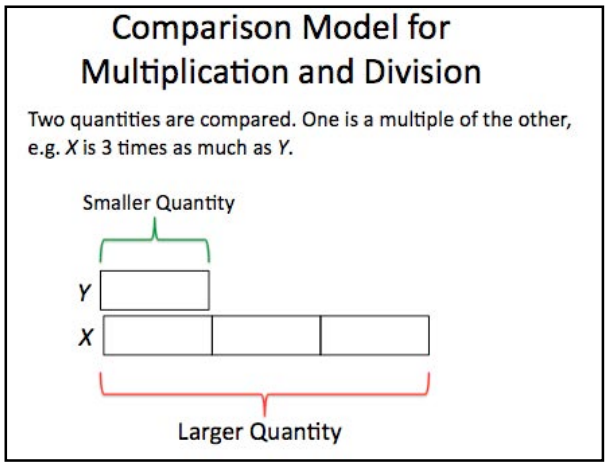
Type 3



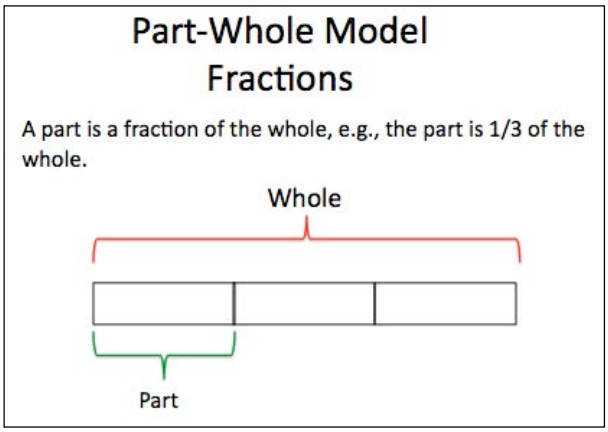
The Part-Whole Model is used to solve problems when the whole is divided into a number of equal parts. The whole, the size of the equal part, or the number of parts can be unknown.

Type 4

The Comparison Model used for multiplicative comparison problems. The smaller quantity, larger quantity or the multiple could be the unknown.



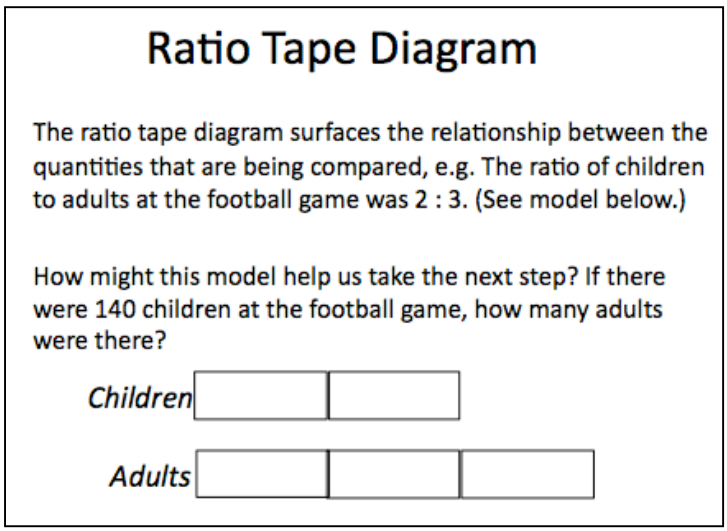
Type 5



The Part-Whole Fraction model is used to model problems involving fractions, e.g. There are 24 flowers in the vase, 1/3 of them are white. How many flowers are white?

Type 6

The Ratio Tape Diagram attaches a clear visual model to the quantities that are being compared. The follow up question will extend students' thinking and computation skills.



Appendices

Name_____
Date_____

Step 1: Read, reread and retell the problem to a shoulder partner in your own words. Be ready to help retell this problem with your class.

Step 2: Rewrite the question in sentence form, leaving a space for the answer.

Step 3: Determine who or what is involved in the problem.

Step 4: Draw and label a unit bar to start the problem.

Step 5: Chunk the problem, adjust the unit bars, and fill in the question mark. Label all your work.

Step 6: Correctly compute and solve the problem.

Step 7: Write the answer is the sentence, and make sure the answer makes sense.

Tape Diagrams and Model Drawing Support the Common Core State Standards

The Standards for Mathematical Practice offer teachers guidance as to how students are expected to engage with the mathematics they are learning. We feel very strongly that the use of tape diagrams while problem solving directly supports active engagement with the following Standards for Mathematical Practice.

Mathematical Practice #1: Make sense of problems and persevere in solving them.

Mathematical Practice #2: Reason abstractly and quantitatively.

Mathematical Practice #4: Model with mathematics

Mathematical Practice #5: Use appropriate tools strategically.

In MTSD teachers will use tape diagram while teaching to the following **content standards**.

Kindergarten

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

Grade 1

Represent and solve problems involving addition and subtraction.

1.OA.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1]

1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Grade 2

Represent and solve problems involving addition and subtraction.

2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Grade 3

Represent and solve problems involving multiplication and division.

3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]

Develop understanding of fractions as numbers.

3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Grade 4

Use the four operations with whole numbers to solve problems.

4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Extend understanding of fraction equivalence and ordering.

4.NF.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Grade 5

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.OA.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Grade 6

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]

6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Grade 7

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Response to Intervention for Mathematics

The IES Practice Guide Assisting Students Struggling with Mathematics: Response to Intervention for Elementary and Middle School published eight recommendations to help guide teachers, schools, and districts as they work to meet the needs of struggling math students.

We strongly believe that tape diagrams and model drawing directly support the implementation of the following recommendations:

Recommendation #2
Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8.

Recommendation #4
Interventions should include instruction on solving word problems that is based on common underlying structures.

Recommendation #5
Intervention materials should include opportunities for students to work with visual representation of mathematical ideas and teachers should be proficient in the use of visual representations of mathematical ideas.

Addition and Subtraction Problem Situations

	Result Unknown	Change Unknown	Start Unknown
Add To (join)	Eight frogs sat on the grass. Three frogs jumped out of the pond and joined them on the grass. How many frogs are on the grass now? 8 + 3 = ☐	Eight frogs were sitting on the grass. Some other frogs jumped over there and joined them. Then there were eleven frogs on the grass. How many frogs jumped over to join them? 8 + ☐ = 11	Some frogs were sitting on the grass. Three other frogs jumped over there to join them. Then there were eleven frogs. How many frogs were on the grass before those three joined them? ☐ + 3 = 11
Take From (separate)	Eleven grapes were in a bowl. I ate eight of the grapes. How many grapes are in the bowl now? 11 – 8 = ☐	Eleven grapes were in a bowl. I ate some of the grapes. Then there were three grapes remaining in the bowl. How many grapes did I eat? 11 – ☐ = 3	A bowl held a bunch of grapes. I ate eight grapes. Then there were three grapes remaining in the bowl. How many grapes were in the bowl before I ate some? ☐ – 8 = 3
	Total Unknown	Addend Unknown[1]	Both Addends Unknown[2]
Put Together or Take Apart	The vase held three red flowers and eight yellow flowers. How many flowers were in the vase? 3 + 8 = □	Eleven flowers are in the vase. Eight are red flowers and the rest are yellow. How many flowers are yellow? 8 + □ = 11 11 – 8 = □	Grandma has eleven flowers in a vase. Some of the flowers are red and some are yellow. How many flowers of each color might she have? 11 = □ + ¯
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	"How many more?" version: Nakeia has eight apps on her iPhone. and Justin has eleven apps on his. How many more apps does Justin have than Nakeia? "How many fewer?" version: Nakeia has eight apps on her iPhone. and Justin has eleven apps on his. How many fewer apps does Nakeia have than Justin? 8 + □ = 11 11 – 8 = □	"More" version with "more": Justin has three more apps on his iPhone than Nakeia has on hers. Nakeia has eight apps. How many apps does Justin have? "Fewer" version: Nakeia has three fewer apps on her iPhone than Justin has on his iPhone. Nakeia has eight apps. How many apps does Justin have? 8 + 3 = □	"More" version: Justin has three more apps on his iPhone than Nakeia has on her iPhone. Justin has eleven apps. How many apps does Nakeia have? "Fewer" version: Nakeia has 3 fewer apps on her iPhone than Justin has on his iPhone. Justin has eleven apps. How many apps does Nakeia have? □ + 3 = 11 11 – 3 = □

In each problem type (shown as a row), any one of the three quantities in the situation can be unknown, leading to the subtypes shown. The various subtypes show important language variants which, while mathematically the same, require separate attention. In the literature on word problems, the descriptions or labels for the problem types vary by using somewhat different names.

[1] Either addend can be unknown; both variations should be included.
[2] Take apart situations with both addends unknown give experience with finding all of the decompositions of a number and reflecting on the patterns involved, and are especially important for numbers within 10. Equations with totals on the left of the equal sign help students understand that the = sign means does not mean "makes" or "results in" but always means "is the same amount as."

Adapted from the *Common Core State Standards for Mathematics* (CCSSM, 2010, p. 88) which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council, 2009, pp. 32-33) and from the draft CCSSM progressions document on K-5 Operations and Algebraic Thinking (2011, p. 7 & 9).

Resources

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National Governors Association Center for Best Practices, Council of Chief State School Officers Title: Common Core State Standards for Mathematics. Publisher: National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C. Copyright Date: 2010.

Thinking Blocks: Model your Math Problems retrieved September 2, 2012 http://www.thinkingblocks.com/

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